Modeling short-term rate in a soft corridor

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Abstract

In an effective corridor based framework, overnight call rate should be constrained between the standing lending and deposit rates. However, in India, there is evidence of call rate often breaching the corridor. Additionally, because of the presence of two lending facility at repo rate and marginal standing facility, there is also a possibility of multiple corridors. This study models this phenomenon as a "soft corridor" by adapting the Jacobi diffusion process in a regime-switching state-space framework. Significant improvement in the likelihood, and lower AIC and BIC supports the hypothesis of the presence of multiple regimes. As a contribution to the state- space estimation literature, study proposes a novel alternative of the Lamperti transform along with the unscented Kalman filter to overcome the non-Gaussian error term of the state process.

Keywords: Overnight rate, Monetary policy, Jacobi-diffusion, Lamperti transform, Non-linear state-space

JEL: E43, G12, G17, H12

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1. Introduction

Under the expectation hypothesis, long term yields are an average of expected future short-term rates. Consequently, short-term rates provide a fundamental building block for most of the popular term-structure models (Vasicek, 1977; Cox et al., 1985). Moreover, the majority of the central banks rely on the short-term interest rates for monetary policy implementation. Hence, the behavior of short-term rate, most importantly, its deviation from the reference rate, provides important insights about the effectiveness of the monetary policy implementation.

Continuous-time diffusion models, where interest rate is defined as a sum of deterministic drift term and a stochastic diffusion term is a mainstay of the short-rate modeling literature (Vasicek, 1977; Cox et al., 1985; Aït-Sahalia, 2002; Choi, 2009; Chan et al., 1992). These models, usually implemented on a monthly or lower frequency, mask the implication of the daily monetary policy operations on the interest rates. On the other hand, theoretical studies (Woodford, 2001; Whitesell, 2006; Bindseil and Jabecki, 2011) exploring the relationship between monetary policy and overnight rate, seldom used in the empirical setting.

This study aims to bridge the gap between these two strands of literature. We adopt the theoretical corridor structure (Bindsei, 2015; Woodford, 2001) with some modification, suitable to the Indian context, into diffusion based modeling framework. In a pure corridor based system, central bank uses two standing facilities to maintain the overnight rate around the policy target rate. The rate at lending facility acts as a ceiling, where financial institutions can borrow overnight from the central bank, and the rate at deposit facility acts as a floor, where they can park their excess cash and earn interest. Most popular diffusion processes, such as Vasicek (Vasicek, 1977) and CIR(Cox et al., 1985), ignore the above-mentioned theoretical characteristics of the overnight rate in the corridor based system. Jacobi diffusion process, as defined in the equation 4, is an attractive alternative to model time series constrained in such corridor. Larsen and Sørensen (2007) and De Jong et al. (2001) utilized this property of Jacobi diffusion process to model exchange rates in the target zone. This study borrows from the exchange rate literature and models the interest rate in the corridor based system as a Jacobi diffusion process.

Further, the Indian monetary policy framework has important structural differences from the pure corridor framework explored in Bindsei (2015). Post-May 2011, with the introduction of the marginal standing facility (MSF), Indian banks have access to two lending facilities. Under liquidity adjustment facility, banks can borrow up-to 0.25% of net demand and time liabilities (NDTL) at the repo rate. In the event of unanticipated large liquidity shock, banks can take recourse to the MSF. Further, the timing of central bank liquidity adjustment facilities and inter-bank trading is also not non-overlapping as assumed in the Bindsei (2015). Hence, banks and central banks both update their belief about the prevailing liquidity situation while observing activities in the inter-bank market and the RBI operations (Patra et al., 2016). As we detail in section 3.1, this information spillover and availability of the two lending rates create a possibility of multiple corridors of the interest rates. The study relies on Markov switching framework proposed by Hamilton (1990) to incorporate the possibility of multiple corridors.

RBI, the central bank of India, places prudent limits on borrowing from the repo facility as well as the marginal standing facility. Also, not all participants of the call money market (such as co-operative banks) have access to the standing and liquidity adjustment facilities. Uncertainty in liquidity projection and effectiveness in managing the liquidity, are two strong impediments of managing interest rates inside the corridor. Patra et al. (2016) and Reddy (2002) have highlighted the uncertainties and challenges in forecasting the liquidity in the Indian banking system. Additionally, Kahn (2010) has argued that there is a stigma attached with borrowing from the central bank using the standing facility, hence commercial bank might agree to borrow from the other commercial banks at higher rates than the marginal lending rate. Because of all these frictions, overnight rate (figure 2) in India seems to breach both sides of the corridor frequently.

It is not possible to incorporate these breaches directly into Jacobi diffusion process, as likelihood would not be defined for the interest rate outside the corridor. To model these breaches, we cast the regime switching diffusion process into a state-space setting. We assume, because of the frictions, interest rates are observed with an error term, and underlying unobserved process follows the Jacobi diffusion process.

To summarize the framework, we model overnight call rate in India as a regime-switching state-space Jacobi diffusion process. This framework derives its structure from the monetary policy framework adopted by the Indian central bank and theoretical insights provided by Woodford (2001); Whitesell (2006); Bindseil and Jabecki (2011). For analyzing interest rate in a corridor based system at the higher frequency, we see this framework as a preferred framework over other statistical diffusion models. Significant improvement in the likelihood, and lower AIC and BIC supports the hypothesis of the presence of multiple regime.

Jacobi process defined by the equation 4 has state-dependent diffusion term, resulting in non-Gaussian conditional distribution of the $X_{t+h}|X_t$ (Gouriéroux and Valéry, 2004). This violates the normality condition required for state-space estimation using Kalman filter. The popular approach of approximating non-Gaussian conditional distribution to a Gaussian distribution by equating first two moments, produces conditionally biased estimator of the unobservable state variables (Chen and Scott, 2003) and may yield inaccurate results (Jacquier et al., 2002; Brigo and Hanzon, 1998).

As a theoretically robust alternative, we first transform the process X_t into Y_t with unit diffusion using Lamperti transform, and re-frame state-space framework with Y_t as a latent factor. As a result, conditional distribution of the new latent factor $(Y_{t+h}|Y_t)$ is Gaussian. Lamperti transform is established as a desired first step in the stochastic differential equations (SDE) estimation literature (Iacus, 2010; Aït-Sahalia, 2002; Durham and Gallant, 2002). However, surprisingly, state-space estimation literature in finance usually prefers Gaussian approximation(Lemke, 2006) over Lamperti transform. Probably, because in most of the cases, Lamperti transform also transforms linear state-space into a non-linear one, which is computationally demanding . With the advancements in computing power, computational advantage of approximate linear state-space estimation is diminished. Even in the term structure literature, theoretically robust non-linear state-space estimation is started to get traction over approximate linear state space estimation (Andreasen et al., 2019). Following the trend of adopting theoretically robust albeit computationally intensive estimation technique, this study utilizes Lamperti transform to modify the non-Gaussian state-pace into a Gaussian one, and then estimate using Unscented Kalman filter. To our knowledge, this study is among the very few (others being few studies from the environmental science (Breinholt et al., 2011; Iversen et al., 2014)) to utilize Lamperti transform in the state space estimation and probably the first in the finance. Moreover, it can also be applied on other popular diffusion process such as Cox et al. (1985), Aït-Sahalia (2002) and Chan et al. (1992). We see this as an important contribution to the estimation literature in finance.

The rest of the paper is organized as follows. In the next section, we provide a brief overview of the Indian monetary policy framework . Section 3 details the theoretical framework of the corridor based monetary system along with rationale of casting Jacobi diffusion process into state-space regime switching framework . Section 4 presents the empirical framework. Section 5 summarizes the data and section 6 provides the estimation results. Finally, section 6 concludes.

2. Monetary Implementation framework in India: Overview

Whitesell (2006) classifies monetary policy frameworks adopted by different central banks in three broad categories. In a reserve based framework, banks have to maintain certain reserve with the central bank on an average basis over a maintenance period. This framework is adopted by Federal Reserve and Bank of Japan. In a pure corridor based framework, central bank uses two standing facilities to maintain overnight rate around policy target rate. The rate at lending facility acts as a ceiling, where financial institution can borrow overnight from the central bank, and the rate at deposit facility acts as a floor, where they can park their excess cash and earn interest.This framework is practiced in Australia, Canada, New Zealand, Sweden, and Switzerland. Finally, In a hybrid system, central bank employs both the corridor system and reserve requirements.

Post liberalization, Indian monetary policy framework has witnessed many transformations (please refer to Patra et al. (2016) and Pacheco and Shiraly (2014) for the chronicle of the evolution of the Indian monetary policy). In line with the worldwide trend of the decline of reserve position doctrine (Bindsei, 2015), India also gradually shifted from reserve based system to the corridor based system. RBI has reduced the cash reserve ratio (CRR) from the peak of 15% in 1992 to the current rate of 4% (Pacheco and Shiraly, 2014). Based on the recommendation of the 2nd Narshimhan Committee (Narasimham, 1998), in April 1999, RBI started interim liquidity adjustment facility (ILAF) and later in June 2000, adopted the full-fledged LAF. For the period between 2000-2011, even after the adoption of LAF, call money rate fluctuated between the repo and reverse repo rate depending on the prevailing liquidity situation. Also, it frequently breached the corridor (Patra et al., 2016).

Against this backdrop, LAF framework was modified in May, 2011. To mitigate the unwarranted volatility in the money market, in the event of unanticipated liquidity shocks, marginal standing facility (MSF) was introduced. At MSF, banks can borrow above the existing repo LAF facility. Revised corridor was defined with repo rate placed in the middle of the corridor, reverse repo rate as a floor and MSF rate as a ceiling. MSF and reverse repo rate was decided to set in reference to the repo rate, making repo rate the single independent policy rate. Again in September,2014, LAF framework was further modified. Under this modified framework, RBI gradually started relying more on term repos for the liquidity management for better monetary policy transmission. Currently, India follows mix of corridor system and reserve requirement with averaging both. Indian banks have to maintain 100% of the specified cash reserve on a fortnight basis and 95% on the daily basis (Circular, 2015). Bindsei (2015) argues that even in a corridor based framework, reserve requirement with averaging provides a buffer against day to day autonomous factor shocks.

3. Theoretical framework

Bindsei (2015) and Woodford (2001) provide theoretical framework for the systematic corridor based monetary system . Under this framework (see figure 1), in the morning at time t_0, based on its assessment of the prevailing overall liquidity situation of the banking system, central bank conducts open market operations. Next at time t, in the inter-bank market, banks trades among themselves to achieve their targeted reserve level. In the afternoon at time T, autonomous factor shocks (ϵ_T) is realized. Finally at the end of the day, depending on their liquidity position, banks need to avail standing lending facility at i_b or deposit facility at i_d. Framework further assumes that central bank has accurate liquidity projections and it targets zero net liquidity in the banking system through market interventions. In this framework, for a risk neutral bank, under no-arbitrage condition, inter bank rate i_t is weighted average of i_d and i_b.

$$i_t = P(\epsilon_T > 0|I_t)i_d + P(\epsilon_T < 0|I_t)i_b \tag{1}$$

[Insert Figure 1 about here]

Patra et al. (2016) and Reddy (2002) have highlighted the uncertainties and challenges in forecasting the liquidity in the banking system. Also, till 2016, RBI preferred the ex ante overall deficit of 1% of banks NDTL (RBI, 2016). Equation 1 can be modified to accommodate the ex ante liquidity target (denoted as L) of the central bank and inaccuracy of the central bank projections (denoted as η_T).

$$i_t = P(L + \eta_T + \epsilon_T > 0|I_t)i_d + P(L + \eta_T + \epsilon_T < 0|I_t)i_b \tag{2}$$

3.1. Case of multiple regime

Post-May 2011, with the introduction of MSF, Indian banks have access to two lending facilities. Under liquidity adjustment facility, bank can borrow upto 0.25% of NDTL at the repo rate. In case of unanticipated large liquidity shock, banks can take recourse to the MSF. This framework is similar to the TARALAC ("target rate limited access") framework explored by Holthausen et al. (2008) and Bindseil and Würtz (2008). Assuming borrowing limit from the repo facility as ψ and denoting repo rate as i^*, under this framework inter-bank interest rate can be expressed as

$$i_t = P(L+\eta_T+\epsilon_T > 0|I_t)i_d+P(-\psi < L+\eta_T+\epsilon_T < 0|I_t)i^*+(P(L+\eta_T+\epsilon_T < -\psi)|I_t)*i_b \tag{3}$$

In India, timing of the central bank liquidity adjustment facilities and inter-bank trading is overlapping. Call market is open from 9:00 am to 5:00 pm on weekdays (RBI, 2016) while RBI also conducts various repo and fine tuning operations throughout the day (RBI, Press release, 2014, 2015). Hence, banks update their belief about the prevailing liquidity situation while observing activities in the inter-bank market and the RBI operations (Patra et al., 2016).

Under the framework described in the equation 3, overnight rate is dependent on the three conditional probability terms. Under this formulation, behaviour of the overnight rate can be classified into three different possible regimes.

1. $i_d <= i_t <= i_b$: Under no information or neutral information about overall end of the day system liquidity $(L + \eta_T + \epsilon_T)$, all three probability term of the equation 3 can be non-zero. Hence, inter-bank rate would be be weighted average of i_d, i^*, i_b
2. $i^* <= i_t <= i_b$: If banks expect overall end of the day system liquidity to be negative $(P(L + \eta_T + \epsilon_T > 0|I_t) = 0)$, inter-bank rate would be weighted average of i^* and i_b. Considering the stated preference for the overall liquidity deficit by the Indian central bank till 2016, this should be the most probable regime during that period.
3. $i_d <= i_t <= i^*$: If banks expect overall end of the day system liquidity to be mildly negative or positive $(P(L + \eta_T + \epsilon_T < -\psi|I_t) = 0)$, inter-bank rate would be weighted average of i^* and i_d.

To highlight as a validation of the multiple regime, during the period 2000-2011, call rate used to toggle between repo and reverserepo rate, depending on the banks' expectation about overall liquidity ($P(L + \eta_T + \epsilon_T > 0|I_t) \approx 1 \implies i_t \approx i_d$ and $P(L + \eta_T + \epsilon_T < 0|I_t) \approx 1 \implies i_t \approx i_b$) (Patra et al., 2016). Considering, we do not observe particular regime directly, Markov regime switching framework (Hamilton, 1990) is natural choice to model such process.

3.2. *Modeling interest rate inside the corridor : Jacobi diffusion process*

In all three regimes described in the section 3.1, in the absence of any other frictions, overnight interest rate is constrained in the fixed corridor. Jacobi diffusion process defined in the equation 4, is an attractive alternative to model time series constrained in [U,L] (Gouriéroux and Valéry, 2004).

$$dX_t = \theta_2(\theta_1 - X_t)dt + \theta_3\sqrt{(U - X_t)(X_t - L)}dW_t \tag{4}$$

Under this process, as X_t approaches any of the two boundaries U and L, diffusion terms approaches zero. Also, with $\theta_2 > 0$ and $L < \theta_1 < U$, drift term is mean reverting (positive for the $X_t < \theta_1$ and negative for the $X_t > \theta_1$). Hence, under certain boundary conditions (derived in the Appendix A), the process is always constrained in [U,L]. Larsen and Sørensen (2007) and De Jong et al. (2001) utilized this property of Jacobi diffusion process to model exchange rates in the target zone. In a term structure literature, Delbaen and Shirakawa (2002) derived the expression of the bond prices, under the assumption of the short rate following Jacobi process.

3.3. *Case of breaching the corridor*

In a perfect and efficient corridor based framework, overnight interest rate would be strictly constrained between lending and borrowing facility rate. for example, EONIA rate (European overnight rate) is strictly restricted inside the corridor of marginal deposit and lending rate (figure 2). One can model these rates using Jacobi diffusion process (Raudaschl, 2012).

[Insert Figure 2 about here]

However, overnight rate (figure 2) in India seems to breach both sides of the corridor frequently. To highlight, during late 2013, when US federal reserve signaled hawkish stance to the market, overnight call rate stayed outside the corridor of repo and reverse repo for a sustained period of time and did even breach the MSF rate (RBI, 2014).

In India, RBI places prudent limits on borrowing from the repo facility as well as marginal standing facility. The purpose of these regulations is to avoid excessive reliance of bank on overnight funding. However, it results in the non zero probability of breaching the corridor (Nath, 2015). Second, not all participants of the call money market (such as Co-operative banks) have access to the standing and liquidity adjustment facilities. Additionally, Kahn (2010) has argued that there is a stigma attached with borrowing from the central bank using the standing facility, hence commercial bank might agree to borrow from the other commercial banks at higher rates than the marginal lending rate.

Because of these frictions, and other inefficiency in the monetary policy implementation, overnight rate (figure 2) in India seems to breach both sides of the corridor frequently. It is not possible to model Indian overnight rates directly as a Jacobi diffusion process, as likelihood would not be defined for the interest rate outside the corridor. To overcome this, study defines underlying unobserved process X_t as a Jacobi diffusion process and observed overnight call rate is sum of X_t and the error term. This state space formulation will allow us to model interest rates outside the theoretical bounds as well.

These above-mentioned theoretical and empirical insights provide motivation for modeling overnight interest rate as regime switching state-space with jacobi diffusion as an underlying process ((RSSS-JD henceforth). In the next section, we provide implementation details of this framework.

4. Modeling framework

In the proposed framework observed interest rate r_t is sum of the latent factor X_t and the error term. Hence, measurement equation of the the RSSS-JD framework is represented as

$$r_t = X_t + \epsilon_t, \; where \; \epsilon_t \sim N(0, \Omega) \tag{5}$$

As highlighted in the section 3.1, we explore three possible regime to model the latent variable X_t

1. Liquidity neutral regime($i_d <= i_t <= i_b$)
2. Liquidity surplus regime ($i_d <= i_t <= i^*$)
3. Liquidity deficit regime ($i^* <= i_t <= i_b$)

In the context of India, lending rate i_b is the MSF rate, deposit rate i_d is the reverse repo rate, limited lending facility rate i^* is the repo rate. For every regime, latent variable X_t follows a Jacobi diffusion process.

$$dX_t = \theta_{2,s_t}(\theta_{1,s_t} - X_t)dt + \theta_{3,s_t}\sqrt{(Us_t - X_t)(X_t - Ls_t)}dW_t \tag{6}$$

Where, s_t the regime index follows a continuous time first order Markov chain with maximum two states depending on the model specification. To cast this continuous SDE into discrete transition equation of the state space framework, This SDE needs to be discretized (Lemke, 2006). However, diffusion term of the Jacobi process is state dependent, resulting in non-Gaussian conditional distribution of the $X_{t+h}|X_t$ (Gouriéroux and Valéry, 2004). This violates the normality condition required for state- space estimation using Kalman filter. The usual approach of approximating non-Gaussian conditional distribution to a Gaussian distribution by equating first two moments, produces conditionally biased estimator of the unobservable state variables (Chen and Scott, 2003) and may yield inaccurate results (Jacquier et al., 2002; Brigo and Hanzon, 1998).

To handle the non-Gaussian conditional distribution of the $X_{t+h}|X_t$, we transform the process X_t into Y_t with unit diffusion using Lamperti transform. Lamperti transform is extensively used in the SDE estimation literature (Iacus, 2010; Aït-Sahalia, 2002). Durham and Gallant (2002) demonstrate the improvement in the accuracy of the estimation, when applying the Lamperti transform. He advocates Lamperti transform of the original process as necessary first step irrespective of the method used in the estimation. Surprisingly, with such theoretical appeal and documented empirical improvement,Lamperti transform is not used in state space estimation in finance. To our knowledge, this study is among the very few (others being studies from the environmental science (Breinholt et al., 2011; Iversen et al., 2014)) to utilize Lamperti transform in the state space estimation and probably the first in the finance.

If X_t follows the Jacobi diffusion process define by the equation 4, transform process Y_t follows the process defined in the equation 8 (Please ref to the Appendix B for the proof of the Lamperti transform)

$$dY_t = \left(\frac{\theta_2(\theta_1 - 0.5(U_t + L_t))}{0.5\theta_3(U_t - L_t)} \frac{1}{\sin(\theta_3 Y_t)} + \frac{-\theta_3^2 + 2\theta_2}{2\theta_3} \frac{1}{\tan(\theta_3 Y_t)} \right) dt + dW_t \tag{7}$$

The new latent factor Y_t has Gaussian error term after the discretization. Considering daily frequency of the observation, we intend to use Euler discretization of the state process. The state equation of the proposed regime-switching state-space framework can be represented as follows.

$$\begin{aligned} Y_t = Y_{t-1} + &\left(\frac{\theta_{1,s_t}\theta_{2,s_t} - 0.5\theta_{2,s_t}(Us_t + Ls_t)}{0.5\theta_{3,s_t}(Us_t - Ls_t)} \frac{1}{\sin(\theta_{3,s_t} Y_{t-1})} + \frac{-\theta_{3,s_t}^2 + 2\theta_{2,s_t}}{2\theta_{3,s_t}} \frac{1}{\tan(\theta_{3,s_t} Y_{t-1})} \right) \Delta t \\ &+ \eta_t \end{aligned} \tag{8}$$

The measurement equation representing r_t as a function of Y_t would be as follows (Appendix B)

$$r_t = 0.5(Us_t + Ls_t - (Us_t - Ls_t)\cos(\theta_{3,s_t} Y_t)) + \epsilon_t \tag{9}$$

where, $\eta_t \sim N\left(0, \sqrt{(\Delta t)}\right)$, $\epsilon_t \sim N(0, \sigma)$ and $E(\eta_t \epsilon_t) = 0$

As a result of the Lamperti transform of the original proces X_t, both the transition and measurement equation in the new state space framework are non-linear , hence standard Kalman filter can not be applied here. We would rely on unscented Kalman filter, a nonlinear variant of the Kalman filter for the estimation. Unscented Kalman filter proposed by Simon J. Julier (1997), emerged as a better alternative to the existing linear approximation technique of the extended Kalman filter for the non-linear state-space estimation (Christoffersen et al., 2014; Simon, 2006). Unscented Kalman filter uses careful sampling (called 'sigma points') around the states to estimate the mean and the variance of the non-linear function. Kim (1994) provides an efficient approximation to incorporate regime switching in a linear state-space framework using Kalman filter. This study uses the same algorithm in an unscented Kalman filter setting to incorporate regime switching in a non-linear state-space framework.

5. Data

Although RBI adopted corridor based liquidity management framework in 2000, the presence of multiple corridors is only evident post-2011, after the implementation of MSF. Since one of the objectives of the study is to explore multiple corridors of the Indian overnight rate, we use daily overnight call rate data from the period May 2011- to Dec 2018 for this study. Call rate is an inter-bank unsecured overnight lending rate and operational target for the RBI. Naturally, call rate is dependent on the repo rate, a policy rate decided by the RBI. post-2011, RBI also changed the width of the corridor multiple times (see figure 2). In a corridor-based system, volatility of the interest rate is assumed linear in the width of the corridor set by standing facilities (Bindseil and Jabecki, 2011). Hence, both the θ_2 and θ_3 in the equation 4 will change its value with the change in these policy variables. To avoid the additional complexity of incorporating this, we adjust the scale and location of call rate by the following formula

$$\textit{Call-scaled} = \frac{\textit{Call rate} \; - \textit{Reverse repo rate}}{\textit{MSF} - \textit{Reverse repo rate}} \tag{10}$$

As a result of this standardization, MSF rate is adjusted to 1 , revrse repo rate to 0 and repo rate to 0.5. This standardization is common in this strand of literature (Larsen and Sørensen, 2007; De Jong et al., 2001) and not very unreasonable considering the slow moving nature of these policy variables. We rely on RBI database for all the data.

6. Estimation results

As an empirical exercise, this study estimates four different specifications of the state space framework. First, we estimate the model with a single regime (Donated as NR), where i_b acts as an upper bound and i_d as a lower bound of the corridor. For the regime-switching specifications, we select two out of the three possible regimes specified in section 4 (Total ${}^3C_2$ specifications). The first specification (R1 henceforth), defined by $[i^*, i_b]$ and $[i_d, i^*]$ as two corridors. Similarly, the second specification (R2 henceforth) is defined by $[i_d, i_b]$ and $[i_d, i^*]$ as two corridors and the third one ((R3 henceforth)) by $[i_d, i_b]$ and $[i^*, i_b]$. Table 1 summarizes the estimation results. All three regime switching models perform statistically better than the single regime model. There is a significant improvement in the likelihood for all the regime switching models. Since Regime switching specifications are not nested inside the NR model, we cannot compare model using likelihood ratio test (Hamilton, 1990). However, both the information criteria, AIC and BIC are significantly lower for the regime switching specifications than the NR specification. Among the regime switching models, R1 specification performs poorly, and R2 performs the best. Surprisingly, R1 has worse RMSE than NR specification.

[Insert Table 1 about here]

To explore the behavior of the in-sample fit further, we segregate the observations in the different bins based on the level of scaled interest rates (MSF=1, reverse repo=0), and analyze the behavior of the estimated errors of the different specifications. Table 2 summarizes these results. As expected, all these specifications perform poorly outside the MSF (> 1) and reverse repo rate(< 0). However, NR specification fits these outliers better than the regime switching specifications and could be the reason of lower overall RMSE of the NR specification over R1. R1, which defines two regimes as a liquidity surplus (corridor = $[i_d, i^*]$) and liquidity deficit (corridor = $[i^*, i_b]$) regime, seems to fit interest rates in the upper corridor $[i^*, i_b]$ and around the repo rate ($(0.4, 0.6]$), better than the R2. However R2, which defines regimes as liquidity neutral (corridor = $[i_d, i_b)$ and liquidity surplus (corridor = $[i_d, i^*]$) regime, fits the interest rates in the lower corridor ($[i_d, i^*]$) better. Interestingly, R2 also seem to fit interest rates below the lower corridor marginally better than the other competing specifications. As it is evident from the figure 2, breach of the lower bound is quite frequent in the Indian setting. Finally, R3, which defines regimes as liquidity neutral (corridor = $[i_d, i_b)$ and liquidity deficit (corridor = $[i^*, i_b]$), performs best in the upper corridor and reasonably well around the repo rate. However, similar to R1 specification, it also does not fit the interest rate around the reverse repo rate and below it. Considering, the ability of different regime specifications to fit different level of the interest rates, it would be interesting to incorporate all three possible regimes in one specification. However, we leave that exercise because of the possible overparameterization.

[**Insert Table 2 about here**]

7. Conclusion

This study makes an important contribution to two different strands of finance literature. As a contribution to the short-rate modeling literature, this study proposes regime switching state space Jacobi diffusion model to fit overnight rate in a "soft corridor". This framework derives its structure from the monetary policy framework adopted by the Indian central bank and theoretical insights provided by Woodford (2001); Whitesell (2006); Bindseil and Jabecki (2011).

In a corridor-based framework, overnight rate should be constrained between standing lending rate and deposit rate. However, Because of the inaccuracy/challenges in the liquidity projections by the central bank (Patra et al., 2016), limits on the amount bank can avail from the standing facilities and other structural frictions, overnight call rate in India seems to breach both sides of the corridor frequently. To model these breaches, we cast the regime switching diffusion process into a state-space setting. We assume, because of these frictions, interest rates are observed with an error term, and the underlying unobserved process follows the Jacobi diffusion process.

The study further postulates, that based on the information available to the banks, interest rates might operate in different corridors with different lower and upper bounds. We explore this hypothesis in a Markov regime-switching framework, with regimes defined as a different specification of the corridor. Significant improvement in log-likelihood and AIC and BIC suggests the merit of incorporating multiple regimes in this framework.

However, casting Jacobi process in a state space framework results in non-Gaussian conditional distribution of the error term of the state process (Gouriéroux and Valéry, 2004). The popular approach of approximating non-Gaussian conditional distribution to a Gaussian distribution by equating first two moments may yield inaccurate results (Jacquier et al., 2002; Brigo and Hanzon, 1998). as a important contribution to the state-space estimation literature in finance, this study uses Lamperti transform to modify the original process into a process with unitary diffusion, and then estimate the modified non-linear state-space using unscented Kalman filter. This combination of Lamperti transform and unscented Kalman filter provide theoretically robust alternative albeit computationally intensive to the currently popular approximate likelihood methods. Moreover, this technique can be used for most of the popular diffusion models discussed in Aït-Sahalia (2002).

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8. Figures

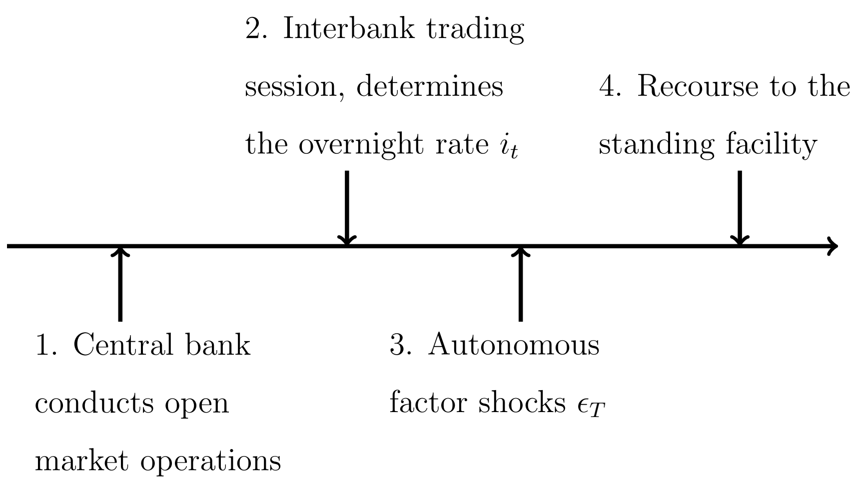

Figure 1: Daily time line of the central bank and the interbank market operations: This figure provides the sequence of event assumed in Bindsei (2015)

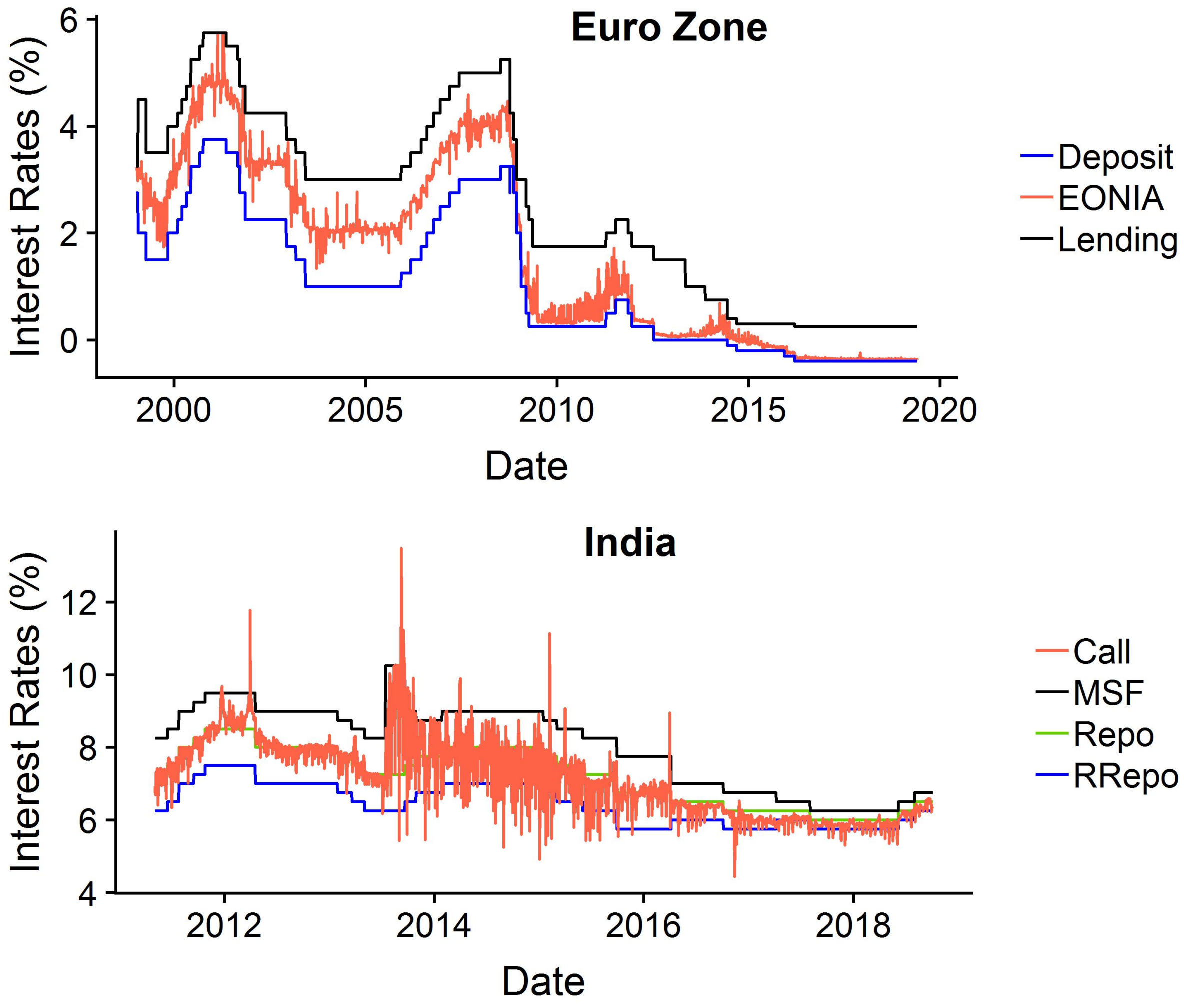

Figure 2: Behavior of the overnight rate: India Vs. Euro-zone: This figure provides comparison between the behavior of the interest rate in the Euro zone (top panel) and India (bottom panel). As it is evident, in the Euro zone EONIA is strictly constrained inside the corridor(case of hard corridor). However, in India, overnight call rate frequently breaches the corridor

9. Tables

Table 1: Estimation results: This table summarizes estimation results of all the specifications explored in this study. Top panel provides the details of regimes for the different specifications. $[\theta_{1,i}, \theta_{2,i}, \theta_{2,i}]$ are the three parameters for the jacobi diffusion process defined in the equation 6. Regime 1 is defined by the corridor $[i_d, i_b]$. Regime 2 is defined by the $[i^*, i_b]$ and regime 3 by $[i_d, i^*]$.

	NR	R1	R2	R3
C1	$[i_d, i_b]$	$[i^*, i_b]$	$[i_d, i_b]$	$[i_d, i_b]$
C2		$[i_d, i^*]$	$[i_d, i^*]$	$[i^*, i_b]$
$\theta_{1,1}$	0.530		0.430	0.505
$\theta_{2,1}$	15.398		2.743	8.012
$\theta_{3,1}$	2.455		1.453	2.152
$\theta_{1,2}$		0.623		0.598
$\theta_{2,2}$		11.949		15.929
$\theta_{3,2}$		0.726		0.885
$\theta_{1,3}$		0.422	0.277	
$\theta_{2,3}$		13.693	12.972	
$\theta_{3,3}$		1.697	1.433	
P11			0.866	0.862
P22		0.672		0.862
P33		0.857	0.681	
RMSE	0.502	0.516	0.366	0.422
Loglik	-59.247	448.677	847.138	663.381
AIC	128.495	-873.355	-1,670.277	-1,302.763
BIC	156.976	-805.000	-1,601.922	-1,234.408

Table 2: RMSE results : This table provides the RMSE of the sample, segregated in the different bins based on the level of scaled interest rates (MSF=1, reverse repo=0)

bins	NR	R1	R2	R3
< 0	0.5	0.88	0.37	0.8
(0,0.2]	0.19	0.32	0.06	0.23
(0.2,0.4]	0.21	0.13	0.08	0.09
(0.4,0.6]	0.24	0.06	0.1	0.06
(0.6,0.8]	0.21	0.11	0.2	0.08
(0.8,1]	0.22	0.31	0.33	0.18
> 1	0.49	0.65	0.71	0.59

Appendix A. Boundary condition for the Jacobi diffusion process

Theorem Appendix A.1. *If X_t follows the diffusion process with drift coefficient $\mu(.)$ and diffusion $\sigma(.)$ in the interval D. let $[a, b] \subset D, a < b$ and process starts at $x \in D$, Let*

$$\psi(x, a, b) = P(X_t \text{ reaches } a \text{ before } b)$$
$$\text{Then, } \psi(x, a, b) = \frac{s(x) - s(a)}{s(b) - s(a)}$$
$$\text{Where, } s(x) \text{ is the scale function defined as}$$
$$s(x) \equiv s(x_0, x) = \int_{x_0}^{x} exp(-I(x_0, z))dz$$
$$\text{and, } I(x, z) = \int_{x}^{z} \frac{2\mu(y)}{\sigma^2(y)}dy$$

Proof. Please refer Bhattacharya and Waymire (2009) Chapter V.9 □

Corollary Appendix A.1.1. *If X_t follows the Jacobi diffusion process as given in the equation 4 and process starts at $x_0 \in (L, U)$, probability of hitting the lower boundary(L) and upper boundary(U) is zero if*

$$\frac{2\theta_2}{\theta_3^2} \times \frac{\theta_1 - L}{U - L} \leq 1 \text{ and, } \frac{2\theta_2}{\theta_3^2} \times \frac{U - \theta_1}{U - L} \leq 1$$

Proof. For the Jacobi diffusion process

$$I(x,z) = \int_x^z \frac{2\mu(y)}{\sigma^2(y)}dy = \int_x^z \frac{2\theta_2(\theta_1 - X_t)}{\theta_3^2(U - X_t)(X_t - L)}dy$$
$$= \frac{2\theta_2}{\theta_3^2} \times \frac{\theta_1 - L}{U - L} \times log\left(\frac{z - L}{x - L}\right) \quad + \quad \frac{2\theta_2}{\theta_3^2} \times \frac{U - \theta_1}{U - L} \times log\left(\frac{z - U}{x - U}\right)$$
$$\left(Assuming,\ \frac{2\theta_2}{\theta_3^2}\frac{\theta_1 - L}{U - L} = p,\ and\ \frac{2\theta_2}{\theta_3^2}\frac{U - \theta_1}{U - L} = q\right)$$
$$I(x,z) = log\left(\frac{(z - L)^p(z - U)^q}{(x - L)^p(x - U)^q}\right)$$
$$Hence,\ \exp(-I(x,z)) = -\frac{(z - L)^p(z - U)^q}{(x - L)^p(x - U)^q}$$

Let x_0 be an arbitrary constant,

$$\implies \exp(-I(x_0,z)) = K(z - L)^p(z - U)^q\ \ Where\ K\ is\ a\ constant$$
$$\implies s(x) \equiv s(x_0,x) = \int_{x_0}^x exp(-I(x_0,z))dz = K\int_{x_0}^x (z - L)^p(z - U)^q dz$$
$$\implies \psi(x,L,U) = \frac{s(x) - s(L)}{s(U) - s(L)} = \frac{\int_x^U (z - L)^p(z - U)^q}{\int_L^U (z - L)^p(z - U)^q}dz$$
$$\implies \psi(x,U,L) = \frac{s(x) - s(L)}{s(U) - s(L)} = \frac{\int_0^x (z - L)^p(z - U)^q}{\int_L^U (z - L)^p(z - U)^q}dz$$

Using the Taylor expansion of the integrand, one can see that $\psi(x,L,U) = 0$ if $p \leq 1$ and $\psi(x,U,L) = 0$ if $p \leq 1$ Which translates into

$$\frac{2\theta_2}{\theta_3^2} \times \frac{\theta_1 - L}{U - L} \leq 1$$
$$\frac{2\theta_2}{\theta_3^2} \times \frac{U - \theta_1}{U - L} \leq 1$$

□

Appendix B. Lamperti transform of the Jacobi diffusion process

Theorem Appendix B.1. *(Iacus, 2010)*

If X_t follows the diffusion process with drift coefficient $\mu(.)$ and diffusion $\sigma(.)$ in the interval D and lamperti transform of X_t is expressed as

$$Y_t = F(X_t) = \int_z^x \frac{1}{\sigma(u)} du$$

Then Y_t, the Lamperti transform of X_t follows SDE process with unitary diffusion expressed as

$$dY_t = \mu^y dt + dW_t$$

Where,

$$\mu^y = \frac{\mu(F^{-1}(y))}{\sigma(F^{-1}(y)))} - \frac{1}{2}\sigma_x(F^{-1}(y))$$

Proof. Using Ito's formula

$$\begin{aligned}
dF(X_t) &= F_X(X_t)dX_t + \frac{1}{2}F_{XX}(dX_t)^2 \\
\implies dF(x_t) &= F_X(X_t)\left(\mu(X)dt + \sigma(X)dW_t\right) + \frac{1}{2}F_{XX}\left(\mu(X)dt + \sigma(X)dW_t\right)^2 \\
&\left(\textit{Replacing } F_X(X_t) = \frac{1}{\sigma(X_t)} \textit{ and } F_{XX}(X_t) = -\frac{\sigma_X(X)}{\sigma(X)^2}\right) \\
\implies dF(X_t) &= \frac{\mu(X)dt + \sigma(X)dW_t}{\sigma(X_t)} - \frac{1}{2}\frac{\sigma_x(X)}{\sigma^2(X)}\sigma^2(X)dt \\
\implies dF(X_t) &= \left(\frac{\mu(X)}{\sigma(X_t)} - \frac{1}{2}\sigma_x(X)\right)dt + dW_t \\
\implies dY_t &= \left(\frac{\mu(F^{-1}(y))}{\sigma(F^{-1}(y)))} - \frac{1}{2}\sigma_x(F^{-1}(y))\right)dt + dW_t
\end{aligned}$$

□

Corollary Appendix B.1.1. *if X_t follows the Jacobi diffusion process as expressed in the equation 4. Y_t, Lamperti transform of X_t follows the stochastic process expressed as*

$$dY_t = \left(\frac{\theta_2(\theta_1 - 0.5(U_t + L_t))}{0.5\theta_3(U_t - L_t)}\frac{1}{\sin(\theta_3 Y_t)} + \frac{-\theta_3^2 + 2\theta_2}{2\theta_3}\frac{1}{\tan(\theta_3 Y_t)}\right)dt + dW_t$$

Proof. Laperti transform for the process X_t

$$Y_t = F(X_t) = \int_{L_t}^x \frac{1}{\theta_3\sqrt{(U_t - X_t)(X_t - L_t)}}dx = \frac{1}{\theta_3}\left(\frac{\pi}{2} + \sin^{-1}\frac{2X_t - (U_t + L_t)}{U_t - L_t}\right)$$

This implies

$$F^{-1}(Y_t) = \frac{1}{2}(U_t + L_t - (U_t - L_t)\cos(\theta_Y))$$

Also,

$$\begin{aligned}
\mu^Y(X_t) &= \frac{\mu(X_t)}{\sigma(X_t)} - \frac{1}{2}\sigma_X(X_t) \\
&= \frac{\theta_2(\theta_1 - X_t)}{\theta_3\sqrt{(U_t - X_t)(X_t - L_t)}} - \frac{\theta_3(L_t + U_t - 2X_t)}{4\sqrt{(U_t - X_t)(X_t - L_t)}} \\
&= \frac{\theta_2(\theta_1 - 0.5(U_t + L_t))}{0.5\theta_3(U_t - L_t)}\frac{1}{\sin(\theta_3 Y_t)} + \frac{-\theta_3^2 + 2\theta_2}{2\theta_3}\frac{1}{\tan(\theta_3 Y_t)} \\
\implies dY_t &= \left(\frac{\theta_2(\theta_1 - 0.5(U_t + L_t))}{0.5\theta_3(U_t - L_t)}\frac{1}{\sin(\theta_3 Y_t)} + \frac{-\theta_3^2 + 2\theta_2}{2\theta_3}\frac{1}{\tan(\theta_3 Y_t)}\right) dt + dW_t
\end{aligned}$$

$\square$