Forecasting Value at Risk and Expected Shortfall with Mixed Data Sampling

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Abstract

I propose applying the Mixed Data Sampling (MIDAS) framework to forecast Value at Risk (VaR) and Expected shortfall (ES). The new methods exploit the serial dependence in short-horizon returns to directly forecast the tail dynamics at the desired horizon. I perform a comprehensive comparison of out-of-sample VaR and ES forecasts with established models for a wide range of financial assets and backtests. The MIDAS-based models significantly outperform traditional GARCH-based forecasts and alternative conditional quantile specifications, especially at multi-day forecast horizons. My analysis advocates models featuring asymmetric conditional quantile and the use of Asymmetric Laplace density to jointly estimate VaR and ES.

Keywords: Mixed Data Sampling (MIDAS), Value at Risk, Expected Shortfall, Backtests, Model Confidence Set

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1. Introduction

The recent 2007-2009 financial crisis has triggered the debate on the accuracy of risk measurement models, especially those focusing on tail risk. Yet, two important grounds remain largely unexplored. First, a voluminous literature study tail risk based on Value at Risk (VaR) estimates, although this measure fails to meet the requirements of a coherent risk metric as defined by Artzner et al. (1999).[1] Among the alternatives, expected shortfall (ES) has recently gained more attention.[2] Despite its importance, there is little empirical works focusing on ES. This is mainly due to the difficulty in estimation and backtesting procedures (Gneiting, 2011). Second, the large extant literature focuses on 1-day ahead risk forecasts, which is clearly insufficient to warn investors and financial institutions and liquidate their positions. As emphasised by Engle (2011), p. 438, *the financial crisis was predicable one day ahead*, and as such, the key failure in risk modelling in financial crisis lies on their deteriorations in multi-day ahead risk forecasts (Brownlees et al., 2011).

This study addresses these gaps by extending the novel quantile regression based on Mixed Data Sampling (MIDAS) of Ghysels et al. (2016) to forecast VaR and ES. The new methods allow for direct forecasting VaR and ES at the desired horizon, while the use of semiparametric specifications avoids making restrictive assumption about conditional return distribution. To the best of my knowledge, this is the first study in the literature that applies MIDAS to obtain ES forecasts. I perform a comprehensive analysis of the forecasting accuracy of the proposed method. The main analysis involves: 43 international indices; three forecast horizons (i.e., 1-day, 5-day and 10-day, respectively); twelve forecasting models; six statistical backtests on both VaR and ES; and an out-of-sample forecast comparison with two loss functions.

My proposal draws on two streams of the literature. First, it is well-established that financial return distribution is not normal and this fact is more pronounced at the multi-day

[1]Previous papers mainly examine the predictive power of risk models in producing VaR forecasts, either explicitly (see, e.g, Berkowitz et al., 2011; Boucher et al., 2014) or implicitly via volatility forecasting (see, e.g, Brownlees et al., 2011; Bams et al., 2017)

[2]The "*Minimum capital requirements for market risk*" of Basel Committee on Banking Supervision (2016) has moved toward using ES, as a complement of VaR, to calculate the regulatory capital requirement. This regulatory agreement is expected to be fully implemented on January 1, 2022.

horizon.[3] Consequently, a good forecasting model at short horizon, such as 1-day ahead, does not necessary yield accurate forecasts at multi-day horizon. Moreover, each quantile in a nonnormal distribution may evolve in different dynamics and depends on different sets of information.[4] These observations suggest that a tail risk model may benefit from a direct estimation of tail area rather than the traditional approach using conditional return distribution models, such as the GARCH family.

Second, several studies document that the dynamic of return volatility is characterised by multiple components capturing information at different time horizons.[5] Given the strong correlation between volatility and return quantiles, it is natural to calibrate a model that could capture different components of information in modelling the tail dynamic. Moreover, Engle (2011) and Neuberger (2012) highlight that long-horizon return distribution depends crucially on the dynamics in short-horizon return process. Therefore, one needs to take into account the serial dependence in short-horizon return when forecasting VaR and ES at the multi-horizon-ahead.

Altogether, I propose to extend the novel MIDAS quantile regression of Ghysels et al. (2016) to directly forecast VaR and ES at the desired horizon. The MIDAS framework introduced by Ghysels et al. (2004) provides an efficient method to link variables sampled at different frequencies. The use of flexible and parsimonious lag polynomials allows MIDAS to directly forecast lower frequency variables by exploiting the data-rich environment at higher frequencies. Thus, MIDAS also provides a suitable framework to capture different components in the tail dynamics by data-driven weighting scheme with flexible shapes. More importantly, this approach offers a direct projection from short-horizon return to multi-horizon return distribution. Andreou et al. (2011) locate the MIDAS approach in the middle of the '*direct*' and '*iterate*' methods in the forecasting literature.[6]

[3]Engle (2011) and Neuberger (2012) find that the asymmetry in return distribution increases with horizon up to one-year and converges very slowly to normality. Recently, Fama and French (2018) apply bootstrapping simulations and document significant return skewness at even 20- and 30-years returns.

[4]Cenesizoglu and Timmermann (2008) and Lima and Meng (2017) document asymmetric effects of economic variables on different parts of the return distribution and time-variation in their explanatory powers.

[5]Some notable examples are Chernov et al. (2003), Corsi (2009) and Engle et al. (2013).

[6]A number of recent studies document the advantage of applying MIDAS in financial forecasts, including Andreou et al. (2013); Foroni et al. (2018) for macroeconomic predictions; Pettenuzzo et al. (2016) for return density; Ghysels et al. (2006) for volatility.

To forecast ES, however, one needs to address its central problem of "*non-elicitability*", which is the lack of a scoring function to facilitate the estimation (Gneiting, 2011). To overcome this issue, I follow two semiparametric approaches proposed in the literature, which directly model VaR and ES and allow their dynamics to vary for each quantile levels. I start from the premise that it is important to account for the serial dependence of higher return process (i.e. daily, in this study) in modelling the conditional density at the desired horizon (Neuberger, 2012). For this purpose, I develop the proposed models on the MIDAS-based quantile regression of Ghysels et al. (2016). In particular, the conditional quantile is based on a mixture of lagged higher frequency returns, which is driven by the data environment and flexibly differs for each quantile level and forecast horizon. Moreover, I also develop an asymmetric specification, which provides better out-of-sample forecast performance than its symmetric counterparts of Ghysels et al. (2016) in most cases.

In the first approach, I adopt the semiparametric model of Taylor (2019) based on the Asymmetric Laplace (AL) density. The author explores the fact that although ES is not individually elicitable, it is jointly elicitable with VaR under a set of suitable scoring functions (Fissler and Ziegel, 2016). Since the AL log-likelihood is a member of this set, VaR and ES can be jointly estimated via maximum likelihood of an AL density. In the second approach, I follow Manganelli and Engle (2004) to combine quantile regression and extreme value theory (EVT). The conditional VaR and ES are estimated by fitting a Generalised Pareto Distribution (GPD) to the extreme observations that exceeded a threshold level.

In the empirical analysis, I employ two alternative semiparametric approaches in the literature as the benchmark methods. First, I consider the filtered historical simulation approach introduced by Barone-Adesi et al. (1999) and Giannopoulos and Tunaru (2005). I use two GARCH models to prefilter the data. VaR and ES forecasts are then obtained from the empirical distribution approximated from simulated paths of returns at the desired horizon using bootstrapping methods. Second, I replace MIDAS-based quantile specifications by the conditional autoregressive VaR (CAViaR) specifications of Engle and Manganelli (2004). The CAViaR-based dynamics have attractive autoregressive structure, yet one needs to form a single-horizon return series that matches the forecast horizon in the model estimation (see, for example, Meng and Taylor, 2018; Taylor, 2019, for applications in VaR and ES forecasts).

I use a battery of statistical tests to compare the out-of-sample VaR and ES forecasts between competing models. In the first stage, I analyse their absolute performance based on the desired properties of VaR and ES as risk metrics, such as correct tail coverage and interdependent exceedances. The backtests include the unconditional coverage test of Kupiec (1995), the dynamic quantile test of Engle and Manganelli (2004), the unconditional ES test on violation residuals of McNeil and Frey (2000), the unconditional and conditional ES test using probability-integral-transform (PIT) of Du and Escanciano (2017) and the multinomial VaR test of Kratz et al. (2018). In the second stage, I investigate the relative performance of competing models in term of minimizing two loss functions. To this end, I form a set of superior models using the Model Confidence Set (MCS) technique of Hansen et al. (2011).

In summary, I obtain strong evidence in favor of the new models across quantile levels and forecasting horizons. Although my focus is to improve multi-day horizon VaR and ES, the MIDAS-based models provide competitive performance to the benchmarks at the 1-day horizon as well. The benefit of MIDAS framework is more pronounced at multi-day forecast horizons. The MIDAS-based models lead to the lowest number of test rejections for both VaR and ES forecasts at the 5- and 10-day horizons. The asymmetric MIDAS-based models generate the lowest forecast errors and are often included in the set of superior models. My empirical results reveal that the naive aggregation to single-horizon return series leads to substantial loss in forecasting information as the CAViaR-based models are inferior to all other models in multi-day forecasting horizons. Finally, I find evidence supporting the joint model of Taylor (2019) that use the AL likelihood in forecasting VaR and ES. The main results are robust when I repeat the analysis for individual stocks, alternative asset classes, different market regimes and separately for developed versus emerging stock markets.

The remainder of the paper is structured as follows. Section 2 introduces my proposed methods. Section 3 reviews the benchmark methods, while Section 4 presents the backtesting procedures for VaR and ES forecasts. Section 5 presents the empirical results on the out-of-sample forecast comparison. Section 6 presents several robustness checks on model performance spanning different market regimes, alternative assets and alternative length of estimation windows. Section 7 concludes the study.

2. Proposed new methods for VaR and ES forecasts

2.1. The MIDAS-based Conditional Quantile

Let $\{r_t\} = ln(P_t/P_{t-1})$ be the daily continuously compounded return series where P_t is the closing price of trading day t. The h-day horizon return is defined as $r_{t,h} = \sum_{i=1}^{h} r_{t+i}$. The h-day VaR of an asset or portfolio returns at the $(1-\alpha)\%$ confidence level is simply the conditional quantile at α, $Q_{\alpha,t-1}(r_{t,h})$.[7] The main ingredient of the proposed models is the MIDAS-based conditional quantile specification introduced by Ghysels et al. (2016). The conditional quantile of returns at any horizon is specified as a linear function of conditioning variables, which can be sampled at different frequencies:

$$Q_{\alpha,t-1}(r_{t,h}) = \beta^0_{\alpha,h} + \beta^1_{\alpha,h} \sum_{d=1}^{D} \varphi_d(\kappa_{\alpha,h})|r_{t-d,1}| \tag{1}$$

where the absolute daily return $|r_{t-d,1}|$ is the conditioning variable with a lag length of D days. $\varphi_d(.)$ is the polynomial function that linearly filters the conditioning variable and projects to the conditional quantile. $\kappa_{\alpha,h}$ is a low-dimensional parameter vector that parsimoniously defines the shape of the filtering function. The vector of estimated parameters $\theta_{\alpha,h} = (\beta^0_{\alpha,h}, \beta^1_{\alpha,h}, \kappa_{\alpha,h})$ is quantile-specific at considered horizon.

A natural extension of (1) is to capture the well-documented asymmetric effects of positive and negative returns (see, e.g. Engle and Manganelli, 2004; Taylor, 2019). The asymmetric conditional quantile can be specified as follow:

$$Q_{\alpha,t-1}(r_{t,h}) = \beta^0_{\alpha,h} + \beta^{1-}_{\alpha,h} \sum_{d=1}^{D} \varphi_d(\kappa_{\alpha,h}) I_{(r_{t-d,1}<0)}|r_{t-d,1}| + \beta^{1+}_{\alpha,h} \sum_{d=1}^{D} \varphi_d(\kappa_{\alpha,h}) I_{(r_{t-d}\geq 0)}|r_{t-d,1}| \tag{2}$$

where $I_{(.)}$ is the indicator function. To retain the parsimonious advantage of the MIDAS framework, I apply one polynomial $\varphi_d(\kappa_{\alpha,h})$ but allowing for different slope coefficients for negative and positive lagged returns. I follow Ghysels et al. (2016) to specify $\varphi_d(\kappa_{\alpha,h})$ by the "Beta" polynomial with two parameters, $\varphi_d(\kappa_1, \kappa_2)$, given that it provides highly flexible shapes (see, Ghysels et al., 2007, for technical discussions and alternative polynomial

[7] Throughout the paper, I use the terms "VaR" and "conditional quantile" at the α quantile level interchangeably to imply the conditional VaR at the $(1-\alpha)$ confidence level. To simplify the notation, I drop the horizon subscript h whenever it does not cause confusions, keeping in mind that the series refers to the h-day horizon from day t to $t+h$.

functions). I restrict $\kappa_1 = 1$ and $\kappa_2 > 1$ in order to have decaying weights on the conditioning variable.[8] The lag length is set at $D = 100$ days for all forecasting horizons. This choice is based on the observation of Ghysels et al. (2006) that using lags longer than 50 days has little effect on volatility forecasts up to 20-days horizon. The conditional quantile is estimated by minimizing the following *tick loss* function:

$$\widehat{\theta}_{\alpha,h} = \underset{\theta_{\alpha,h}}{\text{argmin}} \quad T^{-1} \sum_{t=1}^{T} \left[r_{t,h} - Q_{\alpha,t-1}(r_{t,h})\right] \left[\alpha - I_{(r_{t,h} \leq Q_{\alpha,t-1}(r_{t,h}))}\right] \tag{3}$$

The conditional ES is the expected loss given a VaR violation occurred and can be expressed as:

$$ES_{\alpha,t-1}(r_{t,h}) = E\left[r_{t,h} | r_{t,h} \leq Q_{\alpha,t-1}(r_{t,h})\right] \tag{4}$$

In the next subsections, I present two alternative approaches to estimate ES based on the above MIDAS-based conditional VaR specifications.

2.2. Forecast VaR and ES with Asymmetric Laplace Distribution

In the first approach, I jointly estimate VaR and ES using Asymmetric Laplace (AL) density as proposed by Taylor (2019). This model is motivated by the work of Koenker and Machado (1999), who link the minimisation of the '*tick loss*' function in (3) to the maximum likelihood of an AL density specified as follows:

$$f(r_t) = \frac{\alpha(1-\alpha)}{\sigma} exp\left(-\frac{(r_t - Q_\alpha(r_t))(\alpha - I(r_t \leq Q_\alpha(r_t))}{\sigma}\right) \tag{5}$$

where, for this density, $Q_\alpha(r_t)$ is the time-varying location, while $\sigma > 0$ and $0 < \alpha < 1$ are the scale and skew parameters, respectively. Note that the return process is not assumed to follow AL distribution since the skew parameter α is chosen corresponding to the quantile level of interest. Taylor (2019) argues that if the scale parameter σ varies over time, its maximum likelihood estimation can be interpreted as the time-varying expectation of the

[8] Similar Ghysels et al. (2016), I find that optimising both two parameters can improve the goodness-of-fit in quantile estimate marginally. However, the optimisation comes at significant computational cost and a lower convergence rate.

'*tick loss*' function:

$$\sigma_t = E_{t-1}\left[\left(r_t - Q_\alpha(r_t)\right)\left(\alpha - I\left(r_t - Q_\alpha(r_t)\right)\right)\right] \tag{6}$$

Since Bassett et al. (2004) link the conditional ES to quantile regression by:

$$ES_{\alpha,t-1}(r_t) = E_{t-1}(r_t) - \frac{1}{\alpha}E_{t-1}\left[\left(r_t - Q_\alpha(r_t)\right)\left(\alpha - I\left(r_t - Q_\alpha(r_t)\right)\right)\right]$$

then (6) can be rewritten in term of conditional ES and conditional mean $\mu_t = E_{t-1}(r_t)$ as:

$$\sigma_t = \alpha(\mu_t - ES_{\alpha,t-1}(r_t))$$

Thus, for given specifications on the dynamics of the conditional mean, conditional VaR and ES, the AL density in (5) can be rewritten in conditional terms as:

$$f(r_t) = \frac{1-\alpha}{\mu_t - ES_{\alpha,t-1}(r_t)} exp\left(-\frac{(r_t - Q_{\alpha,t-1}(r_t))(\alpha - I(r_t \leq Q_{\alpha,t-1}(r_t))}{\alpha(\mu_t - ES_{\alpha,t-1}(r_t))}\right) \tag{7}$$

Without the loss of generality, I specify the conditional mean return as an AR(1) formulation where $\mu_t = a_0 + a_1 r_{t-1}$ to account for possible autocorrelation in the return process. I follow Taylor (2019) to specify conditional ES as an exponential function of the conditional quantile:

$$ES_{\alpha,t-1}(r_t) = [1 + \exp(\gamma)]\, Q_{\alpha,t-1}(r_t) \tag{8}$$

where γ controls the joint dynamics of VaR and ES. The use of exponentiation function is to prevent possible crossovers between conditional VaR and ES. The $Q_{\alpha,t-1}(r_t)$ can follow either the MIDAS-based specifications in (1) and (2). Finally, I follow the optimisation procedure of Taylor (2019) to jointly estimate VaR and ES. To assist the optimisation, I separately estimate the coefficients in the conditional mean using maximum likelihood and the conditional quantile using the MIDAS quantile regression.[9] Next, these optimised values are combined with 10^4 randomly sampled candidates for the γ coefficient in the ES formulation to form the vectors of starting parameters. The optimisation is then performed on the negative of the sample log-likelihood of (7). I term the models which define $Q_{\alpha,t-1}(r_t)$

[9]The estimation is based on an R code created by the author following the Matlab toolbox provided by Eric Ghysels

in (1) and (2) as '*Midas-AL*', and '*MidasAs-AL*', respectively.

2.3. Forecast of VaR and ES with Extreme Value Theory

In the second approach, I adopt the two-step estimation procedure suggested by Manganelli and Engle (2004). First, the MIDAS quantile regression of Ghysels et al. (2016) is estimated at a threshold level which is not as extreme as the quantile level of interest. Similar to Manganelli and Engle (2004), I choose the threshold level at $\alpha_u = 7.5\%$. The standardised quantile residuals, Z_{α_u}, are then obtained as follows:

$$Z_{\alpha_u} = \frac{r_t}{Q_{\alpha_u,t-1}(r_t)} - 1 \tag{9}$$

where $Q_{\alpha_u,t-1}(r_t)$ is the conditional quantile at threshold level α_u. Second, I fit the Generalised Pareto Distribution (GPD) to the standardised quantile residuals of threshold violations, i.e $Z^{exceed}_{\alpha_u} = Z_{\alpha_u} | Z_{\alpha_u} > 0 \sim GPD(\widehat{\xi}, \widehat{\varsigma})$, where $\widehat{\xi} < 1$ is the shape parameter and $\widehat{\varsigma}$ is the scale parameter. Conditional VaR and ES at any quantile level $\alpha < \alpha_u$ then can be computed using the results of McNeil and Frey (2000):

$$\begin{aligned}
Q_{\alpha,t-1}(Z_{\alpha_u}) &= \frac{\widehat{\varsigma}}{\widehat{\xi}} \left[\left(\frac{\alpha T}{T_u} \right)^{-\widehat{\xi}} - 1 \right] \\
ES_{\alpha,t-1}(Z_{\alpha_u}) &= Q_{\alpha,t-1}(Z_{\alpha_u}) \left(\frac{1}{1-\widehat{\xi}} + \frac{\widehat{\varsigma}}{(1-\widehat{\xi}) Q_{\alpha,t-1}(Z_{\alpha_u})} \right) \\
Q_{\alpha,t-1}(r_t) &= Q_{\alpha_u,t-1} \left[1 + Q_{\alpha,t-1}(Z_{\alpha_u}) \right] \\
ES_{\alpha,t-1}(r_t) &= Q_{\alpha_u,t-1} \left[1 + ES_{\alpha,t-1}(Z_{\alpha_u}) \right]
\end{aligned}$$

where T_u is the number of exceedances beyond the conditional threshold. In this approach, I denote the model that uses the (1) specification in quantile regression as '*Midas-Evt*', whereas I use the term '*MidasAs-Evt*' when specification in (2) is used.

3. Benchmark Models

In this section, I present a set of benchmark models to examine the predictive power of new methods on out-of-sample VaR and ES forecasts. The details of forecasting models are presented in Table A.1 in Appendix.

3.1. Filtered Historical Simulation

The first benchmark method is the Filtered Historical Simulation (Fhs) introduced by Barone-Adesi et al. (1999) for VaR and extended to ES by Giannopoulos and Tunaru (2005). Kuester et al. (2006) find that this approach outperforms the simple historical simulation as well as the analytical approximation in VaR forecasts.

I consider two GARCH models to prefilter the data, namely the GARCH(1,1) model of Bollerslev (1987) and its asymmetric version, i.e. GJR-GARCH(1,1) model of Glosten et al. (1993). Brownlees et al. (2011) document better volatility forecasting performance for the latter relative to alternative GARCH-type models. To be consistent with the MIDAS-based models, I model conditional mean as an AR(1) process:

$$r_t = a_0 + a_1 r_{t-1} + \sigma_t z_t \tag{10}$$

while the conditional variance process is defined as:

$$\text{GARCH:} \quad \sigma_t^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 \sigma_{t-1}^2 \tag{11}$$

$$\text{GJR-GARCH:} \quad \sigma_t^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 I_{(\varepsilon_{t-1}<0)} \varepsilon_{t-1}^2 + \beta_3 \sigma_{t-1}^2 \tag{12}$$

where $\varepsilon_t = \sigma_t z_t$ is the residuals from the mean equation and z_t is the series of standardised resdiuals, which follows the standardised Skewed Generalised Error (SGE) distribution of Theodossiou (2015), i.e. $z_t \sim SGE(0, 1, \lambda, \eta)$. This distribution allows for tail-fatness and asymmetry in the return process, where the shape parameters $-1 < \lambda < 1$ and $\eta > 0$ control asymmetry and tail thickness, respectively. The distributional density is symmetric when $\lambda = 0$ and skews to the left (right) when $\lambda < 0$ ($\lambda > 0$). When $\lambda = 0$ and $\eta = 2$, it gives the standardised normal distribution (see, e.g., Feunou et al., 2016; Anatolyev and Petukhov, 2016, for application of SGE distribution to financial data).

To estimate conditional VaR and ES at h-day horizon, I perform the following algorithm. First, I randomly sample $\{z_{t+1}^*, z_{t+2}^*, .., z_{t+h}^*\}$ with replacement on day t from the set of standardised residuals. Then, the sampled residuals are plugged into the conditional mean and variance equations (i.e., (10) - (12)) to generate a simulated path of returns $\{r_{t+1}^*, r_{t+2}^*, ..., r_{t+h}^*\}$. The bootstrapped h-day return is then constructed as $r_{t,h}^* = \sum_{i=1}^{h} r_{t+i}^*$. These above steps are repeated $B = 10,000$ times to form an empirical return distribution at the h-day horizon, $\{r^b_{t,h}\} = \{r^1_{t,h}, r^2_{t,h}, ..., r^B_{t,h}\}$. Finally, the conditional VaR is obtained as the α^{th} percentile of the simulated return distribution:

$$Q^B_\alpha(r_{t,h}) = \{r^b_{t,h}\}_{B\alpha} \tag{13}$$

and the corresponding conditional ES is:

$$ES^B_\alpha(r_{t,h}) = \frac{1}{B\alpha}\sum_{b=1}^{B} r^b_{t,h} I_{(r^b_{t,h}<Q^B_\alpha(r_{t,h}))} \tag{14}$$

where $I_{(r^b_{t,h}<Q^B_\alpha(r_{t,h}))}$ is an indicator function. I term the VaR and ES forecasts from this method as "*GARCH-Evt*" when the conditional variance in (11) is used and "*GJR-Fhs*" when (12) is used.

3.2. EVT-based Filter Historical Simulation

An alternative simulation approach is to combine Fhs with EVT as proposed by McNeil and Frey (2000). Novales and Garcia-Jorcano (2018) find that the EVT-based models provide better VaR and ES forecasts than those of non EVT-based models. To this end, I fit a GPD to the in-sample standardised residuals that exceeded the threshold, which correspond to the 7.5% percentile of the standardised residuals. Next, I follow McNeil and Frey (2000) to simulate the conditional return distribution at the h-day horizon using the following algorithm. Similar to the Fhs, I randomly sample $\{z^*_{t+1}, z^*_{t+2}, .., z^*_{t+h}\}$ with replacement from the set of standardised residuals. If the bootstrapped z^* is lower than the threshold level, I replace it with a simulated value from a GPD $(\widehat{\xi},\widehat{\eta})$, where $\widehat{\xi}$ and $\widehat{\eta}$ are the estimated GPD parameters from in-sample standardised residuals. Otherwise, the sampled standardised residuals is used. Then, I obtain the empirical return distribution at the h-day horizon using $B = 10,000$ trails similar to the Fhs algorithm, $\{r^b_{t,h}\} = \{r^1_{t,h}, r^2_{t,h}, ..., r^B_{t,h}\}$. Finally, the conditional VaR and ES are also obtained by (13) and (14) as above. I term the VaR and ES forecasts from this approach "*GARCH-Evt*" and "*GJR-Evt*", depending on whether the filtering model is GARCH(1,1) and GJR-GARCH(1,1), respectively.

3.3. CAViaR-based Models

In the next benchmark method, I replace the MIDAS-based specifications with two analogues drawing from the CAViaR model of Engle and Manganelli (2004):

Symmetric Absolute Value:

$$Q_{\alpha,t-1}(r_t) = \beta_0 + \beta_1 Q_{\alpha,t-2}(r_{t-1}) + \beta_2 |r_{t-1}| \tag{15}$$

Asymmetric Slope:

$$Q_{\alpha,t-1}(r_t) = \beta_0 + \beta_1 Q_{\alpha,t-2}(r_{t-1}) + \beta_2^- I_{(r_{t-1}<0)}|r_{t-1}| + \beta_2^+ I_{(r_{t-1}\geq 0)}|r_{t-1}| \tag{16}$$

The conditional VaR and ES are estimated using either AL density or EVT as described earlier. A contrasting difference between this benchmark method and MIDAS-based models is the treatment of higher frequency observations. The CAViaR model works exclusively on single-horizon setting. This means that one needs to aggregate higher frequency returns to match the target forecasting horizon to perform model estimation. I term the forecasting models '*Sav-AL*' and '*Sav-Evt*' when symmetric absolute value specification is utilised. Alternatively, I refer the models as '*As-AL*' and '*As-Evt*' when the asymmetric slope specification is employed.

4. Evaluation Methods for VaR and ES Forecasts

I employ two alternative ways to evaluate the accuracy of out-of-sample VaR and ES forecasts. First, I assess the absolute performance of VaR and ES forecasts corresponding to their usages as risk measures. Second, I evaluate the relative performance of competing models using two loss functions.

4.1. Absolute Performance Evaluation

4.1.1. VaR backtests

I employ two popular tests to investigate the accuracy of VaR forecasts, including the unconditional coverage (UC) test of Kupiec (1995) and the dynamic quantile (DQ) test of Engle and Manganelli (2004). Under the null hypothesis of the UC test, the number of VaR violations is not statistically different from the chosen quantile level. The test can be performed using the log-likelihood ratio (LR) statistic:

$$LR = 2[T_u ln(T_u/(\alpha T)) + (T - T_u) ln((T - T_u)/(T - \alpha T))]$$

where T is the number of observations, α is the probability level and T_u is the number of VaR exceedances. The LR test statistic follows a $\chi^2(1)$ distribution. Apart from unconditional coverage, DQ further examines the dependence between VaR violations. The test statistic involves a transformation of VaR series to a hit sequence, $Hit_t = I_{(r_t < Q_{\alpha,t-1}(r_t))} - \alpha$. Under the null hypothesis of correct VaR forecasts, Hit_t should have a zero unconditional and conditional expectation given the information set available at time $t-1$. The test can be performed using a linear regression $Hit_t = X\beta + \varepsilon_t$, where X is a set of potential explanatory variables, including a constant, the current level of VaR and five lags of Hit_t. The test statistic is specified as:

$$DQ = \frac{\widehat{b}' X' X \widehat{b}'}{\alpha(1-\alpha)}$$

where $\widehat{b}$ are the estimated coefficients of the linear regression and the DQ test statistic follows $\chi^2(7)$ distribution, where 7 is the column dimension of X.

4.1.2. ES backtests

I consider three backtesting procedures for ES forecasts. First, I employ the discrepancy test of McNeil and Frey (2000). After standardizing by corresponding VaR estimates, the standardised discrepancies between VaR violations and ES forecasts should have unconditional mean of zero under correct risk model. This null hypothesis can be tested using bootstrap method with 10,000 trials as documented in McNeil and Frey (2000).

Second, I adopt the unconditional and conditional ES tests of Du and Escanciano (2017) due to their analogy with the VaR backtests. Instead of explicitly employing ES estimates, they implicitly examine the accuracy of the risk model in tail coverage. These tests are based on the observation that VaR violations should form a class of martingale difference sequence (MDS), indexed by the considered quantile level. Du and Escanciano (2017) argue that the cumulative violations also form MDS and provide meaningful information about the conditional tail when a violation occurs to backtest ES. The cumulative violation process is defined as the integral of VaR violations:

$$H_t(\alpha) = \frac{1}{\alpha} \int_0^{\alpha} h_t(u) \mathrm{d}u$$

where $h_t(u) = I_{(r_t < Q_{u,t-1}(r_t))}$ is the hit indicator at quantile level, u, at time t. If the risk model is correctly specified, $h_t(u)$ has mean u. Similar to Du and Escanciano (2017), I define $u_t = F(r_t|\widehat{\theta}_\alpha, \Omega_{t-1})$ for computational purposes, where $F(.|\Omega_{t-1})$ is the conditional cumulative return distribution given the estimated parameters of the risk model, $\widehat{\theta}_\alpha$. Then, $h_t(u) = I_{(r_t < Q_{u,t-1}(r_t))} = I_{(u_t < u)}$ and the cumulative violations process can be written as:

$$H_t(\alpha, \widehat{\theta}_\alpha) = \frac{1}{\alpha}\int_0^\alpha I_{(u_t<u)}\mathrm{d}u = \frac{1}{\alpha}(\alpha - \widehat{u}_t)I_{(\widehat{u}_t<\alpha)}$$

The unconditional ES test can be conducted by testing the null hypothesis $H_0 : E\left[H_t(\alpha, \widehat{\theta}_\alpha)\right] = \alpha/2$ using a standard t-test:

$$U_{ES} = \frac{\sqrt{T}\left(\overline{H}(\alpha) - \alpha/2\right)}{var(H_t(\alpha))} \sim N(0,1) \tag{17}$$

where T is the number of forecasts and $var(H_t(\alpha)) = \sqrt{\alpha(1/3 - \alpha/4)}$, and $\overline{H}(\alpha)$ is the sample mean of $\{\widehat{H}(\alpha)\}_{t=1}^T$. The conditional ES test can be obtained by checking the null hypothesis being $H_0 : E\left[H_t(\alpha, \widehat{\theta}_\alpha) - \alpha/2|\Omega_{t-1}\right] = 0$. For this purpose, the lag-j autovariance, $\gamma_{T,j}$, and autocorrelation, $\rho_{T,j}$, of $\{H_t(\alpha)\}_{t=1}^T$ for $j \geq 0$ are defined as:

$$\gamma_{T,j} = \frac{1}{T-j}\sum_{t=j+1}^{T}\left[H_t(\alpha) - \alpha/2\right]\left[H_{t-j}(\alpha) - \alpha/2\right] \quad \text{and} \quad \rho_{T,j} = \frac{\gamma_{T,j}}{\gamma_{T,0}}$$

To be consistent with the DQ test, I chose a lag order $m = 5$. The test can then be conducted using a simple Box-Pierce test statistic.

$$C_{ES}(m) = N\sum_{j=1}^{T}\widehat{\rho}_{T,j}^2 \sim \chi_m^2 \tag{18}$$

Finally, I employ the multinomial VaR (MultiVaR) test of Kratz et al. (2018) to evaluate the accuracy in tail coverage by simultaneously testing VaR estimates at multiple quantile levels. From a practical viewpoint, this test has the advantage of not having to store predictive distribution $F(.|\Omega_{t-1})$ from the risk model at each forecast. For a given starting quantile level of interest, α, I consider a series of VaR forecasts at levels $\alpha_1, .., \alpha_N$ given by:

$$\alpha_j = \alpha + \frac{j-1}{N}(1-\alpha), \quad j = 1, .., N \tag{19}$$

I consider the starting quantile level $\alpha = 0.025$, which is equivalent to backtesting ES forecasts at the 2.5% quantile level. This choice is motivated by the requirement of Basel Committee on Banking Supervision (2016) for ES forecasts.[10] The sequence $X_t = \sum_{j=1}^{N} I_{t,j}$ with $I_{t,j} = I_{(r_t < Q_{\alpha_j,t-1})}$ counts the number of VaR estimates being violated at each time t. Similar to the individual VaR estimate, the sequence (X_t) should satisfy the unconditional coverage, i.e. $P(X_t \leq j) = \alpha_{j+1}, j = 0, ..., N$ for all t and the conditional coverage, i.e. X_t is independent of X_s for all $s \neq t$. Kratz et al. (2018) show that the two above conditions can be tested using multinomial distribution MN$(T, (p_0, ..., P_N))$ where T is number of trials. At each trial, I observe $N + 1$ outcomes $(0, 1, ..., N)$ depending on how many VaR levels are breached with corresponding probabilities $p_0, ..., p_N$. The observed cell counts are defined as $O_j = \sum_{t=1}^{T} I_{X_t=j}$. Under the null hypothesis of correct model, the random vector $(O_0, O_1, ..., O_T)$ should follow a multinomial distribution. Kratz et al. (2018) propose several test statistics to examine this hypothesis. In my application, I choose the Nass test (Nass, 1959) with $N = 4$, that exhibits to be a good compromise between size and power of test (for technical details, refer to Kratz et al., 2018).

4.2. Relative Performance Evaluation

To evaluate the relative accuracy and facilitate decision making between different forecasting methods, it is necessary to employ loss functions. Models generate low expected loss arguably preferred over those with higher loss values. To simplify the notation in this subsection, let $\widehat{Q}_t = Q_{\alpha,t-1}(r_t)$ be the conditional VaR and $\widehat{ES}_t = ES_{\alpha,t-1}(r_t)$ be the conditional ES. Since VaR is elicitable using (3), Giacomini and Komunjer (2005) argues that this function is a natural choice to compare VaR forecasts:

$$L_Q(\widehat{Q}_t) = (r_t - \widehat{Q}_t) \left[\alpha - I_{(r_t \leq \widehat{Q}_t)}\right] \tag{20}$$

The recent study of Fissler and Ziegel (2016) suggests a family of strictly consistent loss functions in which VaR and ES forecasts are jointly elicitable. I adapt a member of this family defined in Fissler et al. (2015) to jointly compare the forecast errors of VaR and ES

[10]The use of quantile regressions cannot guard against the possibility of the well-known quantile crossing. On any day when the issue is observed, I apply the recently developed method of monotonically rearrangement of Chernozhukov et al. (2010) to correct the problem.

as:

$$
\begin{aligned}
L_{FZG}(\widehat{Q}_t, \widehat{ES}_t) = {} & (I_{(r_t<\widehat{Q}_t)} - \alpha)\widehat{Q}_t - I_{(r_t<\widehat{Q}_t)} r_t \\
& + \frac{\exp(\widehat{ES}_t)}{1+\exp(\widehat{ES}_t))}\left(\widehat{ES}_t - \widehat{Q}_t + \frac{1}{\alpha} I_{(r_t<\widehat{Q}_t)}(\widehat{Q}_t - r_t)\right) \\
& + ln\left(\frac{2}{1+\exp(\widehat{ES}_t)}\right)
\end{aligned} \tag{21}
$$

Using these loss functions, I apply the model confidence set (MCS) method of Hansen et al. (2011) to form a set of superior models. The MCS procedure starts with the initial set of forecasting models, M_0 to deliver the superior set of models $M^*_{1-\alpha^*}$, which contains smaller number of models, $m^* < M_0$, for a given significant level α^*.[11] In the main analysis, I use $\alpha^* = 5\%$ to construct the 5% MCS.[12] The test applies an elimination rule where at each step, a significance test is conducted to eliminate the worst performing model based on an equivalence test, $\delta_{\mathbb{M}}$, and an elimination rule $e_{\mathbb{M}}$, as follows:

$$
\begin{aligned}
&H_{0,\mathbb{M}} : E(\Delta L_{i,j,t}) = 0, \quad \text{for all} \quad i, j \in \mathbb{M} \\
&H_{A,\mathbb{M}} : E(\Delta L_{i,j,t}) \neq 0, \quad \text{for some} \quad i, j \in \mathbb{M}
\end{aligned}
$$

where $\mathbb{M} \subset M_0$ is the set of remaining models at each step and $\Delta L_{i,j,t}$ is the loss difference between model i and j at time t. If the null hypothesis $H_{0,M}$ is not rejected by the equivalence test $\delta_{\mathbb{M}}$, the MCS is defined as $M^*_{1-\alpha^*} = \mathbb{M}$. Otherwise, the worst performing model is eliminated using the elimination rule $e_{\mathbb{M}}$. I employ the equivalence test based on the range statistic in Hansen et al. (2011):[13]

$$
T_{\mathbb{M}} = \max_{i\in\mathbb{M}} |t_{i,j}| \tag{22}
$$

[11] Note that I use α^* to differentiate the significant level of MCS analysis to the quantile level, α, in VaR and ES forecasts.

[12] The 10% MCS is presented in Table A.11 and provides similar result.

[13] I also employ the alternative test statistic in Hansen et al. (2011), which is the semi-quadratic statistic. The results are presented in Table A.12 and yields similar to the main analysis

where

$$t_{i,j} = \frac{\overline{\Delta L}_{i,j}}{\sqrt{\widehat{Var}(\overline{\Delta L}_{i,j})}} \quad ; \quad \overline{\Delta L}_{i,j} = T^{-1} \sum_{t=1}^{T} \Delta L_{i,j,t}$$

where $\overline{\Delta L}_{i,j}$ is the average sample loss difference between models i and j, $\widehat{Var}(\overline{\Delta L}_{i,j})$ is estimate of the asymptotic variance of $\overline{\Delta L}_{i,j}$, computed using a block-bootstrap with $10,000$ trials and a block size set at $l = 4$ observations.[14]

The elimination rule is then specified as:

$$e_{\mathbb{M}} = \underset{i \in M}{arg\,max} \sup_{j \in \mathbb{M}} t_{i,j} \tag{23}$$

where the model with the highest value of $t_{i,j}$ is eliminated if the null hypothesis is rejected. The test is sequentially repeated until the MCS is reached at a given confidence level.

5. Empirical Results

5.1. Data

I employ daily U.S. dollar-dominated returns for 42 international indices and the MSCI World index. I obtain total return indices of 24 developed markets (DM) from FTSE, and for the 18 emerging markets (EM) indices from the S&P/IFCI database. The series correspond to highly liquid and investable indices, which track real returns for a foreign investor investing in the country's equity market. The sample period is from January 2, 1996 to December 31, 2017 for most of the markets with a total of 5740 days.[15] The full list of countries is provided in Table A.1 in Appendix.

Table 1 reports the descriptive statistics for the index return series. Panel A displays information about the 1-day return horizon, while Panels B and C present the results for 5- and 10-day horizons, respectively. The columns provide the mean and quantiles for the cross-sectional distribution of the statistics presented in rows, including the annualised mean, annualised standard deviation, skewness, kurtosis and the Jarque-Bera statistic. With the

[14]The MCS results with alternative block sizes (2 and 6) or the use stationary bootstrapping in A.13 give similar results

[15]The only two exceptions are Portugal ,which starts on May 04, 1998 and Russia, which starts on April 02, 1997.

only exception of Portugal, all markets have positive mean returns over the sample in all the three horizons. The return series have, on average, negatively skewed and leptokurtic empirical distributions. Notably, average skewness increases in absolute value with horizons which is in line with the findings of Neuberger (2012) and Ghysels et al. (2016). Indeed, the Jarque-Bera statistics strongly reject the null hypothesis of normality for all indices and horizons.

5.2. Estimates of MIDAS-based models

In this paper, I am interested in the VaR and ES forecasts using two commonly used quantiles in the literature, at $\alpha = (0.01, 0.05)$ probability levels, respectively. I consider three forecast horizons: 1-day, 5-day and 10-day. The choice of 1-day horizon allows for direct comparison of my results to the established methods in the literature, which mainly focus on 1-day ahead forecasts. The choice of 10-day horizon is motivated by the baseline horizon used for the capital requirements under the Basel III regulatory agreement.

The main focus of this study is to improve the out-of-sample performance of VaR and ES forecasts using MIDAS-based models. However, the in-sample estimation of the proposed models provide some noteworthy observations. For this purpose, I present the estimated parameters of the MIDAS-based models using the first estimation window of 2500 daily returns. I start by the estimation results for the MSCI world index at $\alpha = 0.05$.[16] Next, I further examine the cross-sectional variations in parameter estimates across countries.

Table 2 presents results for the AL-based models described in Section 2.1. Columns (1) are the results for the *Midas-AL* model, while columns (2) are the results for the *MidasAs-AL* model. The row "*Log-L*" provides the maximised log-likelihood value of AL density presented in (7), while "*Hit*" is the empirical violation rate in the estimation sample.

I observe strong time-variation in the conditional VaR since the slope coefficients $\beta^{1}_{\alpha,h}$ $(\beta^{1-}_{\alpha,h}, \beta^{1+}_{\alpha,h})$ are always statistically significant at conventional levels. The γ coefficient governing the dynamics of conditional ES is also statistically significant across models and horizons. Not surprisingly, the negative and positive returns have different impact on the quantile dynamics, although the asymmetry is less pronounced at longer horizons. For example, the $\beta^{1+}_{\alpha,h}$

[16] The estimation results for $\alpha = 0.01$ provide largely similar conclusions and available upon request.

estimate at the 1-day horizon is -0.354 and not statistically significant, whereas its value at the 10-day horizon is 9.548 and highly significant with the magnitude almost equal to that of $\beta_{\alpha,h}^{1-}$ (-10.842). The *MidasAs-AL* model provides better goodness-of-fit than the symmetric counterpart as shown by the "*Log-L*" values. Finally, the percentages of VaR exceedances are always close to 5%, signalling good tail coverages for both models over the estimation period.

Table 3 reports the estimated parameters for the EVT-based models. Columns (1) correspond to the *Midas-Evt* model, while columns (2) refer to the *MidasAs-Evt* model. I also report the likelihood value of (7) using estimated VaR and ES for comparison purposes, although the estimation of EVT-based models does not involve AL density maximisation. The estimation results are generally in line with those reported in Table 2. The asymmetric effects of lagged returns become less pronounced at longer horizon. Both models have Hit percentages close to 5%. Finally, the likelihood values are only slightly lower than their counterparts in Table 2, which directly maximise the AL likelihood.

Tables 4 and 5 provide a summary of the cross-sectional parameter estimates for the newly proposed models. Some observations are worth noting. First, the coefficients of negative lagged returns ($\beta_{\alpha,h}^{1-}$) have greater magnitude on average than those of lagged positive returns ($\beta_{\alpha,h}^{1+}$). This finding provides evidence of asymmetric effects of lagged returns across countries and forecast horizons. Second, the cross-sectional standard deviation of parameter κ_2 is relatively more pronounced than those of other parameters, particularly at multi-days horizons. Although κ_2 does not have a direct economic interpretation, this observation indicates significant variation in the shapes of the weighting function applied to the lagged conditioning variable. Since I apply the same lag length in all estimations, this finding highlights the flexibility of the MIDAS framework in capturing significant heterogeneity in tail dynamics across market indices and forecasting horizons (see, e.g., Gu and Ibragimov, 2018, for similar evidence of heterogeneity in the tail of international index return using the "Cubic law").

5.3. Out-of-Sample Forecast Evaluation

I now focus on the out-of-sample (OOS) VaR and ES forecasts from the MIDAS-based models and the benchmark models presented in Section 2.2. To this end, I employ a rolling window approach with a fixed length of 2500 daily observations. I estimate the parameters for each model using the most recent 2500 daily observations and obtain VaR and ES forecasts for all quantile levels and for 1-, 5- and 10-day ahead. Then, I move the estimation window 10 days forward and iterate this procedure until I reach the end of the sample. Thus, this procedure yields a total of 324 OOS forecasts, spanning the period from August 2, 2005 to December 30, 2017.

5.3.1. Absolute Forecasting Performance

The results for VaR forecasts of competing models at the 1% and 5% quantile levels are presented in Table 6. Panel A shows the results for the 1-day horizon, whereas Panels B and C display the results for 5- and 10-day forecast horizons, respectively. The first two columns present the empirical hit percentage over the OOS period. For each test, I count the number of model rejections across the countries. Column '*Total*' is the sum of rejections across quantile levels for each test. For example, the value of 3 for the *GARCH-Fhs* model at the 1% quantile in the UC column of Panel A indicates that the 1% VaR forecasts of this model at the 1-day horizon are rejected by UC test in 3 out of 43 indices. Thus, for each forecasting horizon, the best model has the lowest value in each column.

The MIDAS-based models provide competitive results to the benchmark models at the 1-day horizon, but superior results at the 5- and 10-day horizons. All models perform reasonably well in the UC test at 1-day horizons and the levels of hit percentage are close to the quantile level. At longer forecast horizons, however, all benchmark models significantly underestimate the risk, whereas the MIDAS-based models produce the violation rates close to the quantile levels. At 10-day horizon, the two MIDAS-based models with AL density provide the best performance since they are not rejected in any market at both quantile levels.

The results from the DQ test offer three additional insights. First, the asymmetric models often provide smaller number of test rejections than the symmetric alternatives, especially at the 1-day horizon. However, this effect is considerably weaker at the 10-day horizon, which is in line with the in-sample estimates of the previous subsection. Second, the performance of CAViaR-based models deteriorate significantly at the 5- and 10-day forecast horizons. For instance, the 5% VaR forecast of the *As-AL* model is rejected in only 3 out of 43 indices at the 1-day horizon. This number rises remarkably at the 10-day horizon, indicating that the *As-AL* model is rejected in 33 out of 43 markets. Third, the MIDAS-based models consistently provide competitive performance in all three forecasting horizons. In fact, the *Midas-Evt* model has the lowest number of rejections in both the 5- and 10-day forecast horizons. The contrasting performance between MIDAS-based and CAViaR-based models at the multi-day horizon highlights the deficiency of temporal aggregation to match target horizon in VaR forecasts and consistent with the simulation study in Ghysels et al. (2016).

Next, I focus on the result for ES forecasts in Table 7. In the columns, I present evaluation results for the four ES backtests described earlier in Section 2.3. These tests include the discrepancy test of McNeil and Frey (2000) (denoted UES1), the unconditional (UES2) and conditional (CES) tests of Du and Escanciano (2017) and the multi-VaR test of Kratz et al. (2018). Again, for each test, I report the number of model rejections across countries, while column '*Total*' is the sum of this number across quantile levels. Lower number in each column indicates superiority.

The results are generally in line with those in Table 6. First, all models provide acceptable results in two unconditional ES tests with no clear superiority of one model over another. Second, similar to VaR forecasts, the models with asymmetric specification in conditional quantile yield smaller numbers of test rejection. This observation, however, is less pronounced at the 5- and 10-day forecast horizons. Finally, the CAViaR-based models are clearly the worst performing models, whereas the MIDAS-based models are superior at multi-day forecasting horizons. Particularly in the multi-VaR test, all benchmark models are inferior to the new models at 5-day and 10-day horizons[17]. This finding further highlights the benefit of MIDAS framework in exploiting the richness of daily returns to forecast the tail dynamics at multi-day return horizons.

5.3.2. Relative Forecasting Performance

While the absolute performance evaluation is useful to validate the competing models, it provides little insight about their relative performance. Next, I investigate the relative performance of forecasting models based on the two loss functions presented in the previous

[17]The only exception is the *MidasAs-AL* model at 5-day horizon

section. Table 8 reports the average OOS forecast losses for all models under consideration. Panel A shows results for the 1-day horizon, while Panels B and C report results for the 5- and 10-day forecast horizons, respectively. In each panel, I compute the cross-sectional average of the mean forecast losses across the 43 indices using the L_Q and L_{FZG} loss functions, and then report them separately for the 1% and 5% quantile levels. For each column, I highlight cell with the best method.

The most accurate methods often appear in the final two rows, which correspond to the asymmetric MIDAS-based models. The *MidasAs-AL* model yields the most accurate forecasts at the 1% quantile, while the *MidasAs-Evt* is the best model at 5% quantile. The only exception is the 1-day horizon, for which the *GJR-Fhs* model achieves the best performance. The CAViaR-based models also perform well at the 1-day horizon, but their average losses rise significantly at multi-day forecast horizons.

Table 9 presents the MCS results for the L_Q and L_{FZG} loss functions separately for each quantile level and forecast horizons. The entry in each column counts the number of times (out of 43 indices), that the model in row is excluded from the 5% MCS. For example, the entry for L_Q function of the *GARCH-Fhs* model at the 1% quantile level and 1-day horizon is 7. This number indicates that this model is excluded from the MCS in 7 out 43 cases. Therefore, a smaller number indicates superior performance cross-sectionally.

The main findings from the MCS results are following. First, in line with the absolute performance evaluation, there is significant benefit of using asymmetric models at 1-day horizon, but the impact is less pronounced as the forecast horizon gets longer. Second, the *MidasAs-AL* model provides the best overall performance and often be included in the set of superior models in most cases. For example, this model is never excluded from the MCS in all indices at both quantile levels at 10-day forecasting horizon. The GARCH-based models also perform well but are often inferior to the asymmetric MIDAS-based models. Third, the CAViaR-based models perform worst at the multi-day forecast horizon and are often excluded from MCS, especially at the 1% quantile level.

Overall, I obtain promising results for the MIDAS-based models for VaR and ES forecasts. The proposed models consistently belong to the best performing models with low number of rejections across backtests in all quantile levels and forecasting horizons. The new methods also yield the lowest forecast errors and are often included in the set of superior models, especially at forecasting horizons longer than 1-day ahead. In contrast, the alternative models that rely on a single-horizon returns are always inferior to all other models at multi-day forecast horizons. This finding suggests significant benefits of accounting for serial dependence in short-horizon return process to predict the tail dynamics of long-horizon return distribution. Finally, I also find evidence supporting the asymmetric specification in conditional quantile. In terms of ES forecasting method, the jointly model using AL density generally provide better forecasts than the EVT-based alternative.

6. Robustness Checks

6.1. Model Performance and Market Regimes

The accuracy of risk measures is particularly important during periods of financial distress. Thus, I evaluate model performance across different market regimes. Especially, I separate the OOS forecasts into three subsamples: (i) the pre-crisis period from August 2, 2000 to July 31, 2007; (ii) the crisis period from August 1, 2007 to December 31, 2009; (iii) the post-crisis period from January 1, 2010 to December 31, 2017.

Tables A.3 and A.4 in Appendix report the average OOS forecast losses and MCS results for the competing models for each forecasting horizon, quantile level and sub-period. Not surprisingly, the forecast losses increase significantly during the crisis period for all models, quantile levels and forecasting horizons. This finding is in line with the recent result of Kourtis et al. (2016) in volatility forecasting. The MIDAS-based models generate similar forecast losses than GARCH-based models during crisis at 1-day and 5-day horizon, but out-perform the latter at 10-day horizon. During the pre-crisis and post-crisis sub-samples, the MIDAS-based models yield the best performance compared to all other competing models. Consistent with results of the full-sample results, the CAViaR-based forecasts often belong to the worst performing models in all sub-samples and particularly at multi-days horizons. Finally, the *MidasAs-AL* model is often included in the superior set across three sub-samples, where the superiority is more pronounced at multi-day forecasting horizons.

6.2. Alternative Assets

My main results focus on the international equity indices. To provide further evidence, I investigate model performance using alternative assets. To this end, I source stock prices of 20 largest companies globally from the "*Global Top 100 companies by market capitalisation*" report by PricewaterhouseCoopers (PwC) on March 3, 2018. The companies are: Apple, Microsoft, Amazon.com, Tencent, Berkshire Hathaway, JPMorgan Chase, Johnson & Johnson, Exxon Mobile, Bank of America, Royal Dutch Shell, Walmart, Wells Fargo, Intel, Anheuser-Busch InBev, Taiwan Semiconductor, AT&T, Chevron, PetroChina, Novartis. The data is collected from DataStream with the maximum available sample period from January 3, 1997 to December 31, 2017.[18] I also consider two alternative asset classes, including: the Barclays U.S. Aggregate Bond Index from September 29, 2003 to December 31, 2017 as a proxy for the bond class. I also consider the S&P Goldman Sachs Commodity Total Return Index (GSCI) from January 1, 2003 to December 31, 2017 as a proxy for the commodity class. These two indices are investable and track the return of an investor from a fully collateralised portfolio of bonds and commodities. For these two indices, I collect data from the CapitalIQ database.

Table A.5 reports the average OOS forecast losses across the considered assets. In line with the main analysis, the MIDAS-based models provide clearly the best VaR and ES forecasts. The asymmetric models yield slightly lower forecast losses than the symmetric counterparts. This observation is generally in line with the model confidence set results in Table A.6. An interesting observation is that the performance of CAViaR-based models with AL density are not considerably inferior to the GARCH-based models compared to the analysis involving only stock indices.

6.3. Model Performance Between Developed and Emerging Markets

The return distributions in developed and emerging markets are typically characterised by distinct features. Therefore, it is of interest to compare the model performance between two the country groups.

Table A.7 provides the average OOS forecast losses separately for each country group. The forecast losses are substantially higher for the emerging countries in all cases. This

[18]Some stocks have shorter historical length but the first observation is no later than January 1, 2005

observation may be the outcome of more noisy data for the emerging stock markets. Nevertheless, the relative performance between competing models is consistent with the main results. The lowest forecast losses are often recorded in the final two rows, which correspond to the asymmetric MIDAS-based models. The MCS results in Table A.8 indicate that the asymmetric MIDAS-based model with AL density provides the best overall performance in both country groups. Therefore, I conclude that the performance of the new models is robust to different characteristics in the return process.

6.4. Alternative Window Length

The OOS forecasts in the main analysis is conducted using rolling window of 2500 observations. This choice is largely driven by the convergence rates of the CAViaR-based models. The single-horizon setting leads to substantial loss of observation for the model estimation. For example, the CAViaR-based models are optimized using only 250 non-overlapping return observations at the 10-day forecast horizon. However, one may concern that using long estimation windows may give unfair advantage to the MIDAS-based methods, for example, compared to the GARCH-based models. To explore this issue, I repeat our analysis using rolling window of 1,500 and 2,000 observations, respectively. In the former case, I exclude the CAViaR-based models due to their low rates of convergence. Tables A.9 and A.10 in Appendix show that my main conclusions are robust to the length of rolling windows. Notably, the performance of EVT-based models deteriorates remarkably in shorter estimation windows. This observation is not surprising since the numbers of extreme exceptions in these cases are lower, which thereby increases estimation errors and reduces the goodness-of-fit in the GPD estimation.

7. Conclusion

Using the MIDAS framework, I propose new models to directly forecast VaR and ES at the desired horizon and quantile level. The semiparametric approach allows flexible dynamics in different quantile levels and avoid making distributional assumptions. In addition, the MIDAS framework utilises the data-rich environment of higher frequency return process to improve the forecast of the tail dynamics in longer horizon. Using a large cross-section of international stock indices, I examine the predictive performance of the proposed models relative to several popular forecasting models at various quantile levels and forecast horizons. Using a battery of backtesting procedures, I obtain strong evidence in favor of the proposed models, which consistently belong to the best performing methods. The MIDAS framework significantly outperforms the GARCH-based models and the alternative semi-parametric models which rely on single-period quantile regression. Finally, models that incorporate asymmetry in the quantile dynamics, and use of the AL density to jointly estimate VaR and ES, generally provide the best forecasts across quantile levels and return horizons. This result is robust to different market regimes, alternative assets and forecast specifications.

My main analysis focuses on VaR and ES forecast, given their practical importance to financial institutions and regulators. Given the superiority of MIDAS-based models on quantile forecasts, an interesting question for future research is whether the MIDAS framework can also improve return density forecast or equity risk premium using the combination of quantile forecasts. Moreover, several studies document significant explanatory powers of economic variables on conditional return distribution features such as volatility (Engle et al., 2013) or different parts of return density (Cenesizoglu and Timmermann, 2008). Thus, additional information from macroeconomic variables can further improve the forecasts of the tail dynamics. The MIDAS framework provides a suitable setting for incorporating such variables, which typically sampled at different frequencies. I leave such extensions to the future research.

Table 1 Descriptive Statistics of International Indices

This table reports the descriptive statistics for the cross section of log index returns. The columns show the mean and quantiles from the distribution of cross-sectional statistics presented in the rows. Panel A reports the statistics for the 1-day horizon, while Panels B and C show the corresponding statistics for the 5- and 10-day horizon, respectively. The last row in each panel reports the Jaque-Bera test statistics under the null hypothesis of normally distributed in the return series.

	Mean	5%	25%	Median	75%	95%
			Panel A: 1-day horizon			
Mean	0.070	0.024	0.053	0.076	0.086	0.114
Std dev	0.262	0.186	0.222	0.248	0.292	0.410
Skewness	-0.206	-0.736	-0.355	-0.192	-0.069	0.300
Kurtosis	12.305	7.291	9.238	10.848	13.230	22.826
Jarque-Bera	30544.96	4426.63	9074.24	14904.09	25029.66	106849.23
			Panel B: 5-day horizon			
Mean	0.350	0.119	0.265	0.380	0.431	0.569
Std dev	0.631	0.421	0.515	0.598	0.710	1.065
Skewness	-0.472	-0.945	-0.695	-0.566	-0.281	0.141
Kurtosis	9.162	5.383	6.228	7.917	10.363	18.562
Jarque-Bera	3083.68	306.03	526.61	1163.91	2698.50	11688.43
			Panel C: 10-day horizon			
Mean	0.700	0.238	0.530	0.759	0.863	1.137
Std dev	0.855	0.562	0.691	0.810	0.971	1.435
Skewness	-0.521	-1.181	-0.718	-0.517	-0.277	0.078
Kurtosis	7.749	4.433	5.342	6.557	8.915	16.746
Jarque-Bera	909.35	57.40	147.94	316.02	930.40	4735.75

Table 2 Estimation of AL-based Models at 5% quantile for the MSCI World Index

This table provides estimated parameters of two AL-based models under the MIDAS framework for the 5% quantile level for the MSCI World index. The results are presented for 1-, 5- and 10-day return horizons. The parameters are estimated using the first moving window with 2500 observations. Columns (1) are the results for the *Midas-AL* model, while Columns (2) are the results for the *MidasAs-AL* model, which specify the conditional quantile in (1) and (2), respectively. The numbers in parentheses below the estimated parameters are p-values, based on bootstrapped standard errors. For parameter κ_2, the null hypothesis is $\kappa_2 = 1$. The row *Log-L* reports the maximised log-likelihood value of AL distribution described in (7), while the row Hit (%) denotes the percentage of times the VaR is exceeded.

	1-day horizon		5-day horizon		10-day horizon	
Model	(1)	(2)	(1)	(2)	(1)	(2)
$\beta^0_{\alpha,h}$	-0.003	-0.004	-0.012	-0.016	-0.036	-0.045
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
$\beta^1_{\alpha,h}$ $\begin{pmatrix}\beta^{1-}_{\alpha,h} \\ \beta^{1+}_{\alpha,h}\end{pmatrix}$	-1.743	-2.706	-4.265	-7.321	-1.865	-10.842
	(0.000)	(0.000)	(0.000)	(0.000)	(0.007)	(0.000)
		-0.354		0.966		9.548
		(0.049)		(0.088)		(0.000)
κ_2	8.523	7.147	4.968	3.060	20.039	2.613
	(0.000)	(0.000)	(0.000)	(0.011)	(0.034)	(0.000)
γ	-1.064	-1.162	-1.228	-0.959	-0.878	-1.081
	(0.000)	(0.000)	(0.000)	(0.000)	(0.013)	(0.000)
Log-L	7092.82	7179.12	931.39	945.11	388.72	406.47
Hit (%)	4.833	4.750	5.000	5.000	5.000	4.583

Table 3 Estimation of EVT-based Models at 5% quantile for the MSCI World Index

This table provides estimated parameters of two EVT-based models under the MIDAS framework for the 5% quantile level for the MSCI World index. The results are presented for 1-, 5- and 10-day return horizons. The parameters are estimated using the first moving window with 2500 observations. Columns (1) are the results for the *Midas-Evt* model, while Columns (2) are the results for the *MidasAs-Evt* model, which specify the conditional quantile in (1) and (2), respectively. The numbers in parentheses below the estimated parameters are p-values, based on bootstrapped standard errors. For parameter κ_2, the null hypothesis is $\kappa_2 = 1$. The row *Log-L* reports the maximised log-likelihood value of AL distribution described in (7), while the row Hit (%) denotes the percentage of times the VaR is exceeded.

	1-day horizon		5-day horizon		10-day horizon	
Model	(1)	(2)	(1)	(2)	(1)	(2)
$\beta^0_{\alpha,h}$	-0.002	-0.004	-0.011	-0.016	-0.031	-0.033
	(0.001)	(0.000)	(0.003)	(0.000)	(0.003)	(0.000)
$\beta^1_{\alpha,h}$ $\beta^{1-}_{\alpha,h}$	-1.625	-2.726	-3.201	-7.124	-1.375	-9.564
	(0.000)	(0.000)	(0.000)	(0.000)	(0.077)	(0.000)
$\beta^{1+}_{\alpha,h}$		0.035		2.116		6.545
		(0.160)		(0.031)		(0.000)
κ_2	8.608	6.073	5.230	2.777	18.960	2.557
	(0.000)	(0.000)	(0.000)	(0.002)	(0.000)	(0.027)
ξ	0.085	0.185	-0.156	-0.227	0.064	0.053
β	0.349	0.294	0.467	0.520	0.585	0.380
Log-L	7000.13	7166.88	931.06	943.69	386.45	404.36
Hit (%)	5.125	5.250	5.000	5.000	5.417	4.583

Table 4 Cross-sectional Estimates of AL-based Models at the 5% quantile

This table provides the average of estimated parameters across countries of the AL-based models at the 5% quantile level. Results are reported at 1-day, 5-day and 10-day return horizons, respectively. The parameters are estimated using the first moving window of 2500 observations. Columns (1) are the results for the *Midas-AL* model, while Columns (2) are the results for the *MidasAs-AL* model, which specify the conditional quantile in (1) and (2), respectively. The numbers in parentheses display cross-sectional standard deviation of the above parameters.

	1-day horizon		5-day horizon		10-day horizon	
Model	(1)	(2)	(1)	(2)	(1)	(2)
$\beta^0_{\alpha,h}$	-0.006	-0.008	-0.018	-0.021	-0.031	-0.034
	(0.003)	(0.004)	(0.018)	(0.019)	(0.038)	(0.032)
$\beta^1_{\alpha,h}$ $\begin{pmatrix}\beta^{1-}_{\alpha,h}\\ \beta^{1+}_{\alpha,h}\end{pmatrix}$	-1.674	-2.296	-3.908	-5.705	-5.089	-9.298
	(0.278)	(0.380)	(1.555)	(2.541)	(3.232)	(5.675)
		-0.660		-1.678		-0.325
		(0.424)		(2.144)		(6.731)
κ_2	12.606	14.234	21.294	14.901	32.993	12.892
	(7.252)	(7.097)	(52.672)	(24.515)	(65.669)	(24.757)
γ	-0.914	-0.984	-0.959	-0.961	-1.094	-1.196
	(0.185)	(0.182)	(0.228)	(0.218)	(0.356)	(0.377)

Table 5 Cross-sectional Estimates of EVT-based Models at the 5% quantile

This table provides the average of estimated parameters across countries of the Evt-based models at the 5% quantile level. Results are reported at 1-day, 5-day and 10-day return horizons, respectively. The parameters are estimated using the first moving window of 2500 observations. Columns (1) are the results for the *Midas-Evt* model, while Columns (2) are the results for the *MidasAs-Evt* model, which specify the conditional quantile in (1) and (2), respectively. The numbers in parentheses display cross-sectional standard deviation of the above parameters.

	1-day horizon		5-day horizon		10-day horizon	
Model	(1)	(2)	(1)	(2)	(1)	(2)
$\beta^0_{\alpha,h}$	-0.005	-0.006	-0.014	-0.018	-0.021	-0.035
	(0.004)	(0.004)	(0.017)	(0.016)	(0.032)	(0.032)
$\beta^1_{\alpha,h}$ $\beta^{1-}_{\alpha,h}$	-1.436	-2.157	-3.432	-4.708	-4.445	-6.723
	(0.288)	(0.422)	(1.367)	(2.114)	(2.526)	(4.324)
$\beta^{1+}_{\alpha,h}$		-0.450		-1.415		-0.327
		(0.397)		(1.887)		(3.800)
κ_2	11.826	12.823	17.905	19.390	17.246	22.111
	(7.527)	(7.474)	(53.094)	(40.973)	(49.149)	(56.400)
ξ	0.067	0.074	-0.007	0.011	-0.043	0.066
	(0.123)	(0.109)	(0.256)	(0.208)	(0.362)	(0.397)
ς	0.434	0.407	0.497	0.476	0.512	0.423
	(0.057)	(0.048)	(0.172)	(0.138)	(0.197)	(0.208)

Table 6 Results of Out-of-Sample VaR Absolute Forecasting Performance

This table summarises the performance of out-of-sample VaR forecasts across 43 international equity indices. Forecasts are based on rolling window of 2500 observations. Panel A provides the results for the 1-day horizon, while Panels B and C reports the results for the 5- and 10-day forecast horizons, respectively. The columns labelled Hit(%) report the percentage of times the VaR estimates are exceeded. The next six columns display the absolute performance of VaR forecasts, based on the unconditional coverage test (UC) of Kupiec (1995) and the dynamic quantile test (DQ) of Engle and Manganelli (2004). For each test in column, I report the number of test rejections out of 43 indices at 5% significant level. Lower number implies superior performance.

	Hit(%)		UC			DQ		
Models	1%	5%	1%	5%	Total	1%	5%	Total
Panel A: 1-day horizon								
GARCH-Fhs	1.065	5.021	3	1	4	15	20	35
GARCH-Evt	0.997	5.064	0	0	0	14	19	33
GJR-Fhs	1.063	4.966	3	5	8	11	7	18
GJR-Evt	0.996	5.054	2	4	6	4	5	9
Sav-AL	1.074	4.994	3	1	4	21	21	42
Sav-Evt	1.042	5.188	1	1	2	21	24	45
As-AL	1.069	4.908	5	4	9	12	3	15
As-Evt	1.020	5.024	0	3	3	14	5	19
Midas-AL	1.030	4.950	2	1	3	14	21	35
Midas-Evt	1.021	5.119	1	0	1	19	24	43
MidasAs-AL	1.042	4.897	4	4	8	10	4	14
MidasAs-Evt	0.998	4.975	2	4	6	11	5	16
Panel B: 5-day horizon								
GARCH-Fhs	1.529	5.951	7	6	13	15	11	26
GARCH-Evt	1.513	5.920	7	10	17	13	10	23
GJR-Fhs	1.389	5.628	4	4	8	9	7	16
GJR-Evt	1.342	5.520	4	3	7	8	4	12
Sav-AL	1.195	5.030	0	0	0	22	24	46
Sav-Evt	1.267	5.104	0	0	0	16	17	33
As-AL	1.237	4.946	2	4	6	27	15	42
As-Evt	1.269	5.075	1	0	1	19	12	31
Midas-AL	1.023	4.706	1	1	2	11	5	16
Midas-Evt	1.012	4.808	0	1	1	7	4	11
MidasAs-AL	0.924	4.675	1	2	3	12	4	16
MidasAs-Evt	1.015	4.782	0	1	1	8	5	13
Panel C: 10-day horizon								
GARCH-Fhs	1.514	5.692	3	2	5	11	5	16
GARCH-Evt	1.514	5.641	2	2	4	13	3	16
GJR-Fhs	1.181	5.307	0	3	3	5	5	10
GJR-Evt	1.188	5.276	1	3	4	6	5	11
Sav-AL	1.261	6.265	3	3	6	21	34	55
Sav-Evt	1.557	5.579	5	1	6	23	17	40
As-AL	1.329	6.530	3	10	13	15	33	48
As-Evt	1.659	5.548	8	1	9	20	14	34
Midas-AL	1.061	4.811	0	0	0	8	1	9
Midas-Evt	1.079	4.427	0	1	1	7	1	8
MidasAs-AL	0.918	4.913	0	0	0	8	2	10
MidasAs-Evt	1.188	4.676	1	1	2	13	2	15

Table 7 Results of Out-of-Sample ES Absolute Forecasting Performance

This table summarises the performance of out-of-sample ES forecasts across 43 international equity indices. Forecasts are based on rolling window of 2500 observations. Panel A provides results for the 1-day horizon, while Panels B and C reports results for the 5- and 10-day forecast horizons, respectively. The next six columns display the absolute performance of ES forecasts, based on the unconditional ES test of zero discrepancy (UES1) of McNeil and Frey (2000), the unconditional (UES2) and conditional ES (CES) tests of Du and Escanciano (2017), the multi-VaR test of Kratz et al. (2018). For each test in column, I report the number of test rejections out of 43 indices at 5% significant level. Lower number implies superior performance.

	UES1			UES2			CES			
Models	1%	5%	Total	1%	5%	Total	1%	5%	Total	MultiVaR
Panel A: 1-day horizon										
GARCH-Fhs	1	1	2	2	1	3	16	35	51	1
GARCH-Evt	0	0	0	1	1	2	16	33	49	2
GJR-Fhs	1	0	1	1	3	4	7	12	19	4
GJR-Evt	2	1	3	3	2	5	8	12	20	4
Sav-AL	0	1	1	4	0	4	22	42	64	3
Sav-Evt	1	1	2	4	1	5	23	41	64	5
As-AL	1	2	3	8	3	11	11	15	26	4
As-Evt	1	2	3	6	4	10	11	8	19	4
Midas-AL	0	1	1	5	0	5	25	40	65	3
Midas-Evt	1	0	1	4	0	4	20	41	61	3
Midas-AL	0	2	2	4	2	6	11	10	21	1
Midas-Evt	1	2	3	3	2	5	10	10	20	3
Panel B: 5-day horizon horizon										
GARCH-Fhs	1	1	2	3	4	7	1	4	5	8
GARCH-Evt	1	0	1	4	5	9	0	4	4	9
GJR-Fhs	3	2	5	3	3	6	1	4	5	7
GJR-Evt	2	2	4	0	2	2	1	2	3	5
Sav-AL	4	1	5	2	0	2	22	34	56	2
Sav-Evt	0	0	0	0	0	0	7	28	35	2
As-AL	3	3	6	13	0	13	15	13	28	3
As-Evt	2	0	2	3	0	3	11	14	25	4
Midas-AL	0	0	0	1	0	1	5	8	13	1
Midas-Evt	0	0	0	0	0	0	4	4	8	1
Midas-AL	1	0	1	2	1	3	4	5	9	3
Midas-Evt	1	2	3	2	1	3	3	4	7	1
Panel C: 10-day horizon horizon										
GARCH-Fhs	1	0	1	0	3	3	2	2	4	6
GARCH-Evt	1	0	1	0	3	3	3	2	5	4
GJR-Fhs	1	0	1	1	1	2	2	4	6	3
GJR-Evt	0	0	0	2	1	3	2	3	5	3
Sav-AL	3	2	5	9	0	9	16	16	32	7
Sav-Evt	0	1	1	3	2	5	12	13	25	6
As-AL	4	6	10	17	2	19	11	13	24	8
As-Evt	0	1	1	3	2	5	9	10	19	8
Midas-AL	0	1	1	1	0	1	4	4	8	1
Midas-Evt	0	0	0	4	1	5	2	2	4	1
Midas-AL	0	2	2	0	1	1	3	2	5	1
Midas-Evt	0	3	3	6	2	8	3	3	6	1

Table 8 Summary of Out-of-Sample Forecast Losses

This table provides the average out-of-sample forecast losses at the 1% and 5% quantile levels for 1-, 5- and 10-day horizons, respectively. L_Q denotes the quantile loss function of (20) and L_{FZG} is the FZG loss function of Fissler et al. (2015) given in (21). The L_Q and L_{FZG} values are multiplied by 10^4 and 10^3, respectively, to facilitate presentation. Lower values correspond to superior performance. Bold numbers indicate best methods in each column.

	Panel A: 1-day horizon				Panel B: 5-day horizon				Panel C: 10-day horizon			
	1%		5%		1%		5%		1%		5%	
Models	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}
GARCH-Fhs	0.489	2.447	1.684	1.821	1.285	6.278	4.138	4.387	1.701	8.147	5.735	5.988
GARCH-Evt	0.490	2.448	1.684	1.821	1.274	6.223	4.126	4.373	1.698	8.132	5.735	5.988
GJR-Fhs	**0.483**	**2.415**	1.668	1.804	1.268	6.192	4.144	4.391	1.717	8.213	5.778	6.031
GJR-Evt	0.484	2.416	1.667	1.803	1.252	6.115	4.121	4.367	1.715	8.201	5.782	6.036
Sav-AL	0.500	2.498	1.702	1.840	1.469	7.146	4.473	4.743	2.240	10.708	6.420	6.733
Sav-Evt	0.500	2.499	1.703	1.842	1.437	6.999	4.418	4.684	2.146	10.862	6.203	6.661
As-AL	0.489	2.445	1.670	1.806	1.394	6.790	4.424	4.692	2.184	10.424	6.492	6.813
As-Evt	0.484	2.420	1.668	1.804	1.369	6.682	4.336	4.597	2.071	10.662	6.118	6.570
Midas-AL	0.497	2.487	1.701	1.839	1.275	6.215	4.187	4.438	1.736	8.311	5.884	6.144
Midas-Evt	0.498	2.489	1.700	1.839	1.284	6.282	4.141	4.390	1.763	8.635	5.691	5.985
MidasAs-AL	0.485	2.426	1.670	1.806	**1.230**	**5.992**	4.143	4.391	**1.621**	**7.759**	5.708	**5.963**
MidasAs-Evt	0.485	2.424	**1.667**	**1.803**	1.263	6.215	**4.087**	**4.336**	1.757	8.665	**5.677**	5.979

Table 9 Model Confidence Set Results

This table reports the results of the 5% Model Confidence Set (MCS) at the 1% and 5% quantile levels for 1-, 5- and 10-day forecast horizons, respectively. The entry in each column presents the number of times out of 43 indices, that the model in row is excluded from the 5% MCS. L_Q denotes the quantile loss function of (20) and L_{FZG} is the FZG loss function of Fissler et al. (2015) given in (21). The range statistic in (22) is used to the equivalence test of the MCS. Lower values corresponds to superior performance. Bold numbers indicate best methods in each column.

	1-day horizon				5-day horizon				10-day horizon			
	1%		5%		1%		5%		1%		5%	
Models	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}
GARCH-Fhs	7	7	15	13	3	3	1	**0**	3	4	2	2
GARCH-Evt	7	6	13	13	3	3	1	**0**	3	4	1	1
GJR-Fhs	2	2	2	2	2	2	1	**0**	6	7	1	1
GJR-Evt	2	2	2	2	2	2	1	**0**	6	7	1	1
Sav-AL	8	8	19	18	9	8	7	7	19	19	6	7
Sav-Evt	11	9	21	20	9	9	7	5	9	16	5	6
As-AL	5	5	**0**	**0**	7	7	5	4	17	18	7	7
As-Evt	4	4	1	1	6	6	2	2	8	17	1	3
Midas-AL	7	7	18	18	4	4	4	3	7	7	5	4
Midas-Evt	10	9	18	17	5	5	1	1	5	8	0	1
MidasAs-AL	**0**	**0**	2	2	**1**	**1**	1	1	**0**	**0**	**0**	**0**
MidasAs-Evt	4	4	3	3	2	2	**0**	**0**	3	7	**0**	**0**

References

Anatolyev, S. and Petukhov, A. (2016). Uncovering the skewness news impact curve. *Journal of Financial Econometrics*, 14(4):746–771.

Andreou, E., Ghysels, E., and Kourtellos, A. (2011). Forecasting with Mixed-Frequency Data. In Clements, M. P. and Hendry, D. F., editors, *The Oxford Handbook of Economic Forecasting*, pages 1–24.

Andreou, E., Ghysels, E., and Kourtellos, A. (2013). Should Macroeconomic Forecasters Use Daily Financial Data and How? *Journal of Business and Economic Statistics*, 31(2):240–251.

Artzner, P., Delbaen, F., Eber, J.-M., and Heath, D. (1999). Coherent Measures of risk. *Mathematical Finance*, 9(3):203–228.

Bams, D., Blanchard, G., and Lehnert, T. (2017). Volatility measures and Value-at-Risk. *International Journal of Forecasting*, 33(4):848–863.

Barone-Adesi, G., Giannopoulos, K., and Vosper, L. (1999). VaR without correlations for portfolios of derivative securities. *Journal of Futures Markets*, 19(5):583–602.

Basel Committee on Banking Supervision (2016). Minimum capital requirements for market risk. Technical report.

Bassett, G. W., Koenker, R., and Kordas, G. (2004). Pessimistic Portfolio Allocation and Choquet Expected Utility. *Journal of Financial Econometrics*, 2(4):477–492.

Berkowitz, J., Christoffersen, P., and Pelletier, D. (2011). Evaluating Value-at-Risk Models with Desk-Level Data. *Management Science*, 57(12):2213–2227.

Bollerslev, T. (1987). A Conditionally Hetroskedastic Time Series Model for Speculative Prices and Rates of Return. *Review of Economics and Statistics*, 69(3):542–547.

Boucher, C. M., Daníelsson, J., Kouontchou, P. S., and Maillet, B. B. (2014). Risk Models-at-risk. *Journal of Banking and Finance*, 44:72–92.

Brownlees, C. T., Engle, R. F., and Kelly, B. T. (2011). A Practical Guide to Volatility Forecasting through Calm and Storm. *The Journal of Risk*, 14(2):3–22.

Cenesizoglu, T. and Timmermann, A. G. (2008). Is the Distribution of Stock Returns Predictable? *Working Papers*.

Chernov, M., Gallant, A. R., Ghysels, E., and Tauchen, G. (2003). Alternative models for stock price dynamics. *Journal of Econometrics*, 116(1-2):225–257.

Chernozhukov, V., Fernandez-Val, I., and Galichon, A. (2010). Quantile and Probability Curves Without Crossing. *Econometrica*, 78(3):1093–1125.

Corsi, F. (2009). A simple approximate long-memory model of realized volatility. *Journal of Financial Econometrics*, 7(5):174–196.

Du, Z. and Escanciano, J. C. (2017). Backtesting Expected Shortfall : Accounting for Tail Risk. *Management Science*, 63(04):940–958.

Engle, R. F. (2011). Long-term skewness and systemic risk. *Journal of Financial Econometrics*, 9(3):437–468.

Engle, R. F., Ghysels, E., and Sohn, B. (2013). Stock Market Volatility and Macroeconomic Fundamentals. *Review of Economics and Statistics*, 95(3):776–797.

Engle, R. F. and Manganelli, S. (2004). CAViaR: Conditional autoregressive value at risk by regression quantiles. *Journal of Business and Economic Statistics*, 22(4):367–381.

Fama, E. F. and French, K. R. (2018). Long-Horizon Returns. *Review of Asset Pricing Studies*, 8(2):232–252.

Feunou, B., Jahan-Parvar, M. R., and Tédongap, R. (2016). Which parametric model for conditional skewness? *European Journal of Finance*, 22(13):1237–1271.

Fissler, T. and Ziegel, J. F. (2016). Higher order elicitability and osband's principle. *Annals of Statistics*, 44(4):1680–1707.

Fissler, T., Ziegel, J. F., and Gneiting, T. (2015). Expected Shortfall is jointly elicitable with Value at Risk - Implications for backtesting. *Risk*, pages 1–7.

Foroni, C., Guérin, P., and Marcellino, M. (2018). Using low frequency information for predicting high frequency variables. *International Journal of Forecasting*, 34(4):774–787.

Ghysels, E., Plazzi, A., and Valkanov, R. (2016). Why Invest in Emerging Markets? The Role of Conditional Return Asymmetry. *Journal of Finance*, 71(5):2145–2192.

Ghysels, E., Santa-Clara, P., and Valkanov, R. (2004). The MIDAS touch: mixed data sampling regressions.

Ghysels, E., Santa-Clara, P., and Valkanov, R. (2006). Predicting volatility: Getting the most out of return data sampled at different frequencies. *Journal of Econometrics*, 131(1-2):59–95.

Ghysels, E., Sinko, A., and Valkanov, R. (2007). MIDAS regressions: Further results and new directions. *Econometric Reviews*, 26(1):53–90.

Giacomini, R. and Komunjer, I. (2005). Evaluation and Combination of Conditional Quantile Forecasts. *Journal of Business & Economic Statistics*, 23(4):416–431.

Giannopoulos, K. and Tunaru, R. (2005). Coherent risk measures under filtered historical simulation. *Journal of Banking and Finance*, 29(4):979–996.

Glosten, L. R., Jagannathan, R., and Runkle, D. E. (1993). On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks. *Journal of Finance*, 48(5):1779–1801.

Gneiting, T. (2011). Making and evaluating point forecasts. *Journal of the American Statistical Association*, 106(494):746–762.

Gu, Z. and Ibragimov, R. (2018). The “Cubic Law of the Stock Returns” in emerging markets. *Journal of Empirical Finance*, 46(September 2017):182–190.

Hansen, P. R., Lunde, A., and Nason, J. M. (2011). The Model Confidence Set. *Econometrica*, 79(2):453–497.

Koenker, R. and Machado, J. A. F. (1999). Goodness of Fit and Related Inference Processes for Quantile Regression. *Journal of the American Statistical Association*, 94(448):1296–1310.

Kourtis, A., Markellos, R. N., and Symeonidis, L. (2016). An International Comparison of Implied, Realized, and GARCH Volatility Forecasts. *Journal of Futures Markets*, 36(12):1164–1193.

Kratz, M., Lok, Y. H., and McNeil, A. J. (2018). Multinomial VaR backtests: A simple implicit approach to backtesting expected shortfall. *Journal of Banking and Finance*, 88:393–407.

Kuester, K., Mittnik, S., and Paolella, M. S. (2006). Value-at-risk prediction: A comparison of alternative strategies. *Journal of Financial Econometrics*, 4(1):53–89.

Kupiec, P. H. (1995). Techniques for Verifying the Accuracy of Risk Measurement Models. *Journal of Derivatives*, 3(2):73–84.

Lima, L. R. and Meng, F. (2017). Out-of-Sample Return Predictability: A Quantile Combination Approach. *Journal of Applied Econometrics*, 32(4):877–895.

Manganelli, S. and Engle, R. F. (2004). A Comparison of Value-at-Risk Models in Finance. In Szego, G., editor, *Risk Measures for the 21st Century*, pages 123–144. Wiley.

McNeil, A. J. and Frey, R. (2000). Estimation of tail-related risk measures for heteroscedastic financial time series: an extreme value approach. *Journal of Empirical Finance*, 7(3-4):271–300.

Meng, X. and Taylor, J. W. (2018). An approximate long-memory range-based approach for value at risk estimation. *International Journal of Forecasting*, 34(3):377–388.

Nass, C. (1959). The $\chi 2$ Test for Small Expectations in Contingency Tables , with Special Reference to Accidents and Absenteeism. *Biometrika*, 46(3):365–385.

Neuberger, A. (2012). Realized skewness. *Review of Financial Studies*, 25(11):3424–3455.

Novales, A. and Garcia-Jorcano, L. (2018). Backtesting extreme value theory models of expected shortfall. *Quantitative Finance*, 7688:1–27.

Pettenuzzo, D., Timmermann, A. G., and Valkanov, R. (2016). A MIDAS approach to modeling first and second moment dynamics. *Journal of Econometrics*, 193(2):315–334.

Taylor, J. W. (2019). Forecasting Value at Risk and Expected Shortfall Using a Semiparametric Approach Based on the Asymmetric Laplace Distribution. *Journal of Business and Economic Statistics*, 37(1):1–13.

Theodossiou, P. (2015). Skewed Generalized Error Distribution of Financial Assets and Option Pricing. *Multinational Finance Journal*, 19(4):223–266.

Appendix A. ONLINE APPENDIX

A.1. List of Index and Forecasting Models

Table A.1 List of VaR and ES Forecasting Models

This table summarizes the competing forecasting models for VaR and ES under consideration.

Abbreviation	Description
Benchmark Models	
GARCH-Fhs	VaR and ES are extracted from the GARCH model of Bollerslev (1987), assuming a SGE distribution (Theodossiou, 2015) for daily returns. Empirical distribution is approximated using filter historical simulation with 10,000 trials.
GARCH-Evt	VaR and ES are extracted from the GARCH model of Bollerslev (1987), assuming a SGE distribution (Theodossiou, 2015) for daily returns. Empirical distribution is approximated by combining filter historical simulation and EVT with 10,000 trials.
GJR-Fhs	VaR and ES are extracted from the GJR-GARCH model of Glosten et al. (1993), assuming a SGE distribution (Theodossiou, 2015) for daily returns. Empirical distribution is approximated using filter historical simulation with 10,000 trials.
GJR-Evt	VaR and ES are extracted from the GJR-GARCH model of Glosten et al. (1993), assuming a SGE distribution (Theodossiou, 2015) for daily returns. Empirical distribution is approximated by combining filter historical simulation and EVT with 10,000 trials.
Sav-AL	VaR and ES are jointly estimated using maximum likelihood of AL density. VaR follows symmetric absolute value specification in (15), while ES dynamic follows specification in (8).
Sav-Evt	Conditional quantile at threshold level of 7.5% is estimated using CAViaR model with symmetric absolute value specification in (15). VaR and ES are jointly computed using the results of McNeil and Frey (2000).
As-AL	VaR and ES are jointly estimated using maximum likelihood of AL density. VaR follows asymmetric slope specification in (16), while ES dynamic follows specification in (8).
As-Evt	Conditional quantile at threshold level of 7.5% is estimated using CAViaR model with asymmetric slope specification in (16). VaR and ES are jointly computed using the results of McNeil and Frey (2000).
New Models	
Midas-AL	VaR and ES are jointly estimated using maximum likelihood of AL density. VaR follows MIDAS-based symmetric absolute value specification in (1), while ES dynamic follows specification in (8).
Midas-Evt	Conditional quantile at threshold level of 7.5% is estimated using MIDAS quantile regression with symmetric absolute value specification in (1). VaR and ES are jointly computed using the results of McNeil and Frey (2000).
MidasAs-AL	VaR and ES are jointly estimated using maximum likelihood of AL density. VaR follows MIDAS-based asymmetric slope specification in (2), while ES dynamic follows specification in (8).
MidasAs-Evt	Conditional quantile at threshold level of 7.5% is estimated using MIDAS quantile regression with asymmetric slope specification in (2). VaR and ES are jointly computed using the results of McNeil and Frey (2000).

Table A.2 List of Country Index and Sources

	Country	Source
World	World Portfolio	MSCI
Developed Markets	Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, The Netherlands, Hongkong, Ireland, Israel, Italia, Japan, South Korea, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, United Kingdom, United States	FTSE
Emerging Markets	Brazil, Chile, China, Czech Republic, Hungary, India, Indonesia, Malaysia, Mexico, Pakistan, Peru, Philippines, Poland, Russia, South Africa, Taiwan, Thailand, Turkey	S&P/IFCI

Table A.3 Out-of-Sample Forecast Losses - Different Market Regimes

This table provides the average out-of-sample forecast losses for different market regimes at the 1% and 5% quantile levels for 1-, 5- and 10-day horizons, respectively. L_Q denotes the quantile loss function of (20) and L_{FZG} is the FZG loss function of Fissler et al. (2015) given in (21). The L_Q and L_{FZG} values are multiplied by 10^4 and 10^3, respectively, to facilitate presentation. Lower values correspond to superior performance. Bold numbers indicate best methods in each column.

	1-day horizon				5-day horizon				10-day horizon			
	1%		5%		1%		5%		1%		5%	
	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}
Panel A: 02/08/2000 - 31/07/2007 (Pre-crisis Subsample)												
GARCH-Fhs	0.426	2.095	1.431	1.507	1.260	5.993	4.049	4.102	1.760	8.101	*5.404*	*5.274*
GARCH-Evt	0.425	2.092	1.430	1.505	1.237	5.882	3.985	4.033	1.765	8.126	5.410	5.281
GJR-Fhs	**0.416**	**2.047**	1.418	1.492	1.248	5.932	4.069	4.123	1.793	8.255	5.491	5.366
GJR-Evt	0.417	2.051	**1.417**	**1.491**	1.221	5.801	3.988	4.036	1.794	8.266	5.489	5.365
Sav-AL	0.436	2.147	1.449	1.527	1.252	5.950	4.119	4.178	1.824	8.385	5.714	5.605
Sav-Evt	0.437	2.152	1.452	1.529	1.273	6.068	4.071	4.126	1.840	9.019	5.564	5.524
As-AL	0.422	2.074	1.425	1.500	1.254	5.962	4.140	4.202	2.027	9.401	6.054	5.979
As-Evt	0.421	2.071	1.423	1.498	1.273	6.065	4.095	4.152	2.170	10.823	5.904	6.022
Midas-AL	0.433	2.131	1.448	1.525	1.137	5.384	3.989	4.036	1.602	7.353	5.608	5.485
Midas-Evt	0.432	2.127	1.447	1.524	1.224	5.814	3.976	4.023	1.666	7.818	**5.394**	**5.319**

Table A.3 ***(continued)***

	1-day horizon				5-day horizon				10-day horizon			
	1%		5%		1%		5%		1%		5%	
	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}
MidasAs-AL	0.417	2.051	1.425	1.500	**1.120**	**5.302**	4.001	4.049	**1.559**	**7.137**	5.638	5.528
MidasAs-Evt	0.421	2.070	1.422	1.497	1.194	5.671	**3.938**	**3.982**	1.789	8.377	5.452	5.349
Panel B: 01/08/2007 - 31/12/2009 (Crisis Subsample)												
GARCH-Fhs	0.721	3.612	2.553	2.784	1.873	9.195	6.079	6.539	2.317	11.128	8.310	8.840
GARCH-Evt	0.720	3.609	2.555	2.786	1.855	9.105	**6.053**	**6.511**	**2.300**	**11.050**	8.307	8.836
GJR-Fhs	0.711	3.563	**2.530**	**2.759**	1.839	9.119	6.131	6.590	2.421	11.580	8.541	9.079
GJR-Evt	0.710	3.558	2.532	2.761	**1.806**	**8.869**	6.081	6.537	2.413	11.536	8.535	9.071
Sav-AL	0.749	3.755	2.591	2.825	2.585	12.525	7.441	7.984	4.016	19.246	11.042	11.801
Sav-Evt	0.751	3.761	2.597	2.832	2.512	12.187	7.244	7.771	3.969	19.472	10.367	11.106
As-AL	0.729	3.653	2.545	2.776	2.283	11.110	7.178	7.713	3.679	17.588	10.980	11.765
As-Evt	**0.709**	**3.555**	2.538	2.768	2.235	10.884	6.855	7.356	3.478	17.141	9.706	10.420
Midas-AL	0.744	3.727	2.588	2.822	1.897	9.409	6.316	6.786	2.563	12.262	8.808	9.369
Midas-Evt	0.745	3.733	2.590	2.825	1.923	9.406	6.200	6.663	2.679	13.119	8.506	9.090
MidasAs-AL	0.724	3.627	2.542	2.773	1.830	8.983	6.093	6.603	2.412	11.506	**8.288**	**8.794**
MidasAs-Evt	0.712	3.570	2.537	2.766	1.897	9.312	6.069	6.529	2.652	12.938	8.524	9.099

Table A.3 ***(continued)***

	1-day horizon				5-day horizon				10-day horizon			
	1%		5%		1%		5%		1%		5%	
	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}
Panel C: 01/01/2010 - 31/12/2017 (Post-crisis Subsample)												
GARCH-Fhs	0.435	2.179	1.481	1.605	1.113	5.465	3.575	3.806	1.496	7.236	5.027	5.289
GARCH-Evt	0.435	2.183	1.481	1.605	1.107	5.434	3.579	3.811	1.495	7.231	5.028	5.290
GJR-Fhs	0.430	2.156	1.467	1.589	1.104	5.413	3.563	3.794	1.484	7.172	5.003	5.263
GJR-Evt	0.430	2.157	1.465	1.587	1.093	5.361	3.563	3.793	1.482	7.165	5.012	5.272
Sav-AL	0.439	2.203	1.493	1.617	1.185	5.810	3.659	3.897	1.804	8.694	5.189	5.469
Sav-Evt	0.439	2.201	1.493	1.617	1.151	5.654	3.647	3.885	1.666	8.698	5.092	5.586
As-AL	0.433	2.170	1.464	1.586	1.160	5.689	3.660	3.897	1.771	8.514	5.235	5.512
As-Evt	0.431	2.162	1.463	1.585	1.131	5.565	3.635	3.871	1.619	8.659	5.075	5.529
Midas-AL	0.438	2.198	1.492	1.617	1.111	5.452	3.591	3.824	1.517	7.340	5.057	5.320
Midas-Evt	0.439	2.200	1.491	1.616	1.106	5.457	3.561	3.794	1.508	7.469	4.904	5.199
MidasAs-AL	**0.430**	**2.154**	1.465	1.587	**1.064**	**5.218**	3.573	3.804	**1.396**	**6.754**	4.907	**5.159**
MidasAs-Evt	0.431	2.163	**1.462**	**1.584**	1.089	5.414	**3.524**	**3.758**	1.476	7.432	**4.859**	5.177

Table A.4 Model Confidence Set - Different Market Regimes

This table reports the results of the 5% Model Confidence Set (MCS) for different market regimes at the 1% and 5% quantile levels for 1-, 5- and 10-day forecast horizons, respectively. The entry in each column presents the number of times out of 43 indices, that the model in row is excluded from the 5% MCS. L_Q denotes the quantile loss function of (20) and L_{FZG} is the FZG loss function of Fissler et al. (2015) given in (21). The range statistic in (22) is used to the equivalence test of the MCS. Lower values corresponds to superior performance.

	1-day horizon				5-day horizon				10-day horizon			
	1%		5%		1%		5%		1%		5%	
	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}
Panel A: 02/08/2000 - 31/07/2008 (Pre-crisis Subsample)												
GARCH-Fhs	6	6	6	6	6	7	2	2	20	18	4	5
GARCH-Evt	7	6	3	3	3	4	1	1	20	18	4	5
GJR-Fhs	1	1	0	0	11	10	3	3	32	30	9	11
GJR-Evt	2	2	0	0	10	10	3	3	33	31	10	11
Sav-AL	14	14	10	10	17	17	4	4	32	32	11	12
Sav-Evt	15	15	9	9	13	14	3	4	22	21	7	8
As-AL	5	5	3	3	14	14	5	5	33	32	10	12
As-Evt	3	4	2	2	13	13	4	4	31	30	8	12
Midas-AL	13	13	9	9	11	12	6	7	18	18	8	9
Midas-Evt	13	13	7	6	14	15	1	1	24	23	6	7
MidasAs-AL	7	7	5	5	5	6	3	3	13	13	5	6
MidasAs-Evt	4	4	1	1	7	7	1	1	24	25	4	4
Panel B: 01/08/2008 - 31/12/2009 (Crisis Subsample)												
GARCH-Fhs	3	3	2	2	5	5	1	1	10	11	2	2
GARCH-Evt	4	4	3	4	5	5	1	1	9	9	2	2
GJR-Fhs	2	2	1	1	4	4	2	1	12	13	2	2
GJR-Evt	2	3	1	1	4	4	2	2	12	13	2	2
Sav-AL	5	5	2	2	20	19	6	6	15	16	4	4
Sav-Evt	5	5	4	4	19	18	8	8	11	12	4	4
As-AL	2	3	2	2	11	11	6	6	15	17	4	5
As-Evt	4	5	1	1	8	7	6	6	10	12	2	2
Midas-AL	3	3	3	3	11	12	1	1	11	11	4	4
Midas-Evt	4	4	3	4	13	13	4	4	11	12	2	2
MidasAs-AL	2	2	1	1	3	3	2	2	3	3	2	2
MidasAs-Evt	4	5	3	3	7	7	1	1	8	9	2	2
Panel C: 01/01/2010 - 31/12/2017 (Crisis Subsample)												
GARCH-Fhs	2	3	11	11	7	7	2	2	5	8	3	1
GARCH-Evt	2	2	12	13	7	7	2	2	5	6	3	1
GJR-Fhs	0	0	2	3	5	5	3	3	15	15	3	1
GJR-Evt	3	3	4	4	5	5	3	3	15	15	3	1
Sav-AL	7	7	19	19	18	17	5	6	34	33	6	6
Sav-Evt	4	4	17	17	11	11	4	4	21	28	6	4
As-AL	4	4	2	3	17	16	7	7	33	33	8	6
As-Evt	2	2	5	5	11	11	3	3	24	31	4	4
Midas-AL	6	6	18	18	10	10	5	6	15	15	7	5
Midas-Evt	6	6	16	16	11	11	1	1	15	14	0	0
MidasAs-AL	3	3	2	3	1	2	2	2	2	2	2	0
MidasAs-Evt	1	2	5	5	9	9	1	1	11	17	1	0

Table A.5 Out-of-Sample Forecast Losses - Alternative Assets

This table provides the average out-of-sample forecast losses for alternative assets at the 1% and 5% quantile levels for 1-, 5- and 10-day horizons, respectively. The alternative assets are defined in Section 4.2. L_Q denotes the quantile loss function of 20 and L_{FZG} is the FZG loss function of Fissler et al. (2015) given in (21). The L_Q and L_{FZG} values are multiplied by 10^4 and 10^3, respectively, to facilitate presentation. Lower values correspond to superior performance. Bold numbers indicate best methods in each column.

	Panel A: 1-day horizon				Panel B: 5-day horizon				Panel C: 10-day horizon			
	1%		5%		1%		5%		1%		5%	
Models	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}
GARCH-Fhs	0.522	2.594	1.701	1.827	1.073	5.172	3.736	3.904	1.442	6.763	5.127	5.235
GARCH-Evt	0.524	2.605	1.702	1.828	1.078	5.195	3.722	3.890	1.464	6.887	5.106	5.212
GJR-Fhs	0.523	2.601	1.691	1.816	1.096	5.274	3.734	3.902	1.478	6.960	5.128	5.236
GJR-Evt	0.517	2.568	1.691	1.817	1.098	5.294	3.730	3.898	1.473	6.931	5.126	5.237
Sav-AL	0.517	2.558	1.672	1.796	1.151	5.538	3.740	3.908	1.645	7.742	5.267	5.387
Sav-Evt	0.521	2.589	1.695	1.821	1.139	5.496	3.842	4.018	1.760	8.611	5.538	5.726
As-AL	0.515	2.541	**1.648**	**1.770**	1.082	5.221	3.706	3.873	1.561	7.376	5.145	5.255
As-Evt	0.518	2.575	1.684	1.809	1.112	5.390	3.830	4.008	1.626	8.095	5.307	5.538
Midas-AL	0.513	2.548	1.686	1.811	1.044	5.037	3.597	3.756	1.376	6.494	4.917	5.021
Midas-Evt	0.519	2.580	1.693	1.818	1.067	5.144	3.656	3.818	1.436	6.980	5.011	5.139
MidasAs-AL	**0.508**	**2.523**	1.668	1.792	**1.035**	**5.004**	**3.565**	**3.721**	**1.324**	**6.272**	**4.836**	**4.937**
MidasAs-Evt	0.517	2.568	1.680	1.804	1.042	5.032	3.630	3.790	1.432	6.987	5.013	5.176

Table A.6 Model Confidence Set Result - Alternative Assets

This table reports the results of the 5% Model Confidence Set (MCS) for alternative assets at the 1% and 5% quantile levels for 1-, 5- and 10-day forecast horizons, respectively. The alternative assets are defined in Section 4.2. The entry in each column presents the number of times out of 43 indices, that the model in row is excluded from the 5% MCS. L_Q denotes the quantile loss function of (20) and L_{FZG} is the FZG loss function of Fissler et al. (2015) given in (21). The range statistic in (22) is used to the equivalence test of the MCS. Lower values corresponds to superior performance.

	Panel A: 1-day horizon				Panel B: 5-day horizon				Panel C: 10-day horizon			
	1%		5%		1%		5%		1%		5%	
Models	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}
GARCH-Fhs	4	4	15	15	3	5	3	3	9	10	3	3
GARCH-Evt	4	4	15	15	3	4	3	3	11	11	3	3
GJR-Fhs	2	2	13	13	3	5	4	3	13	12	3	3
GJR-Evt	2	2	15	15	5	6	3	3	13	13	3	3
Sav-AL	2	2	6	6	9	9	1	1	10	8	3	3
Sav-Evt	4	4	14	14	9	10	7	7	17	17	7	8
As-AL	1	1	4	4	3	5	2	2	9	7	2	2
As-Evt	2	2	14	14	5	7	4	3	13	12	3	3
Midas-AL	1	1	12	12	3	3	0	0	8	7	1	1
Midas-Evt	3	3	14	15	4	4	2	2	14	13	1	2
MidasAs-AL	0	0	7	8	2	2	1	1	7	5	0	0
MidasAs-Evt	1	1	12	12	4	6	1	1	7	8	1	1

A.4. Robustness Checks: Different Country Groups

Table A.7 Out-of-Sample Forecast Losses - Different Country Groups

This table provides the average out-of-sample forecast losses for different country groups at the 1% and 5% quantile levels for 1-, 5- and 10-day horizons, respectively. L_Q denotes the quantile loss function of 20 and L_{FZG} is the FZG loss function of Fissler et al. (2015) given in (21). The L_Q and L_{FZG} values are multiplied by 10^4 and 10^3, respectively, to facilitate presentation. Lower values correspond to superior performance. Bold numbers indicate best methods in each column.

	1-day horizon				5-day horizon				10-day horizon			
	1%		5%		1%		5%		1%		5%	
	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}
Panel A: Developed Stock Markets												
GARCH-Fhs	0.465	2.330	1.625	1.760	1.218	5.972	3.878	4.124	1.573	7.580	5.415	5.677
GARCH-Evt	0.465	2.329	1.625	1.760	1.203	5.896	3.874	4.119	1.574	7.584	5.419	5.682
GJR-Fhs	0.463	2.318	1.610	1.743	1.214	5.947	3.891	4.137	1.608	7.730	5.485	5.750
GJR-Evt	0.463	2.317	1.610	1.743	1.191	5.834	3.869	4.114	1.602	7.702	5.487	5.752
Sav-AL	0.472	2.362	1.635	1.771	1.378	6.726	4.157	4.421	2.146	10.291	5.972	6.287
Sav-Evt	0.473	2.370	1.636	1.772	1.344	6.567	4.114	4.375	1.990	10.146	5.780	6.261
As-AL	0.467	2.337	1.608	1.741	1.310	6.401	4.136	4.402	2.082	9.993	6.146	6.479
As-Evt	**0.462**	**2.314**	1.606	1.739	1.292	6.314	4.057	4.314	1.952	9.935	5.716	6.132
Midas-AL	0.470	2.352	1.634	1.770	1.199	5.864	3.927	4.175	1.582	7.613	5.527	5.795
Midas-Evt	0.471	2.359	1.633	1.769	1.212	5.941	3.885	4.131	1.627	7.902	5.324	5.599
MidasAs-AL	0.465	2.328	1.608	1.741	**1.160**	**5.676**	3.872	4.117	**1.461**	**7.043**	**5.323**	**5.582**
MidasAs-Evt	0.462	2.315	**1.605**	**1.738**	1.202	5.912	**3.833**	**4.078**	1.612	7.920	5.341	5.636
Panel B: Emerging Stock Markets												
GARCH-Fhs	0.523	2.611	1.767	1.907	1.382	6.722	4.492	4.743	1.838	8.753	6.126	6.363
GARCH-Evt	0.524	2.615	1.767	1.908	1.374	6.686	4.465	4.715	1.829	8.711	6.120	6.358
GJR-Fhs	**0.512**	**2.554**	1.751	1.890	1.348	6.559	4.493	4.742	1.835	8.736	6.135	6.372
GJR-Evt	0.512	2.556	**1.750**	**1.888**	1.341	6.524	4.464	4.711	1.839	8.754	6.145	6.382
Sav-AL	0.537	2.682	1.793	1.936	1.589	7.694	4.894	5.172	2.345	11.170	6.960	7.263
Sav-Evt	0.536	2.673	1.796	1.939	1.559	7.567	4.827	5.100	2.327	11.674	6.719	7.142
As-AL	0.520	2.593	1.759	1.898	1.504	7.292	4.811	5.082	2.256	10.712	6.873	7.173
As-Evt	0.514	2.567	1.756	1.895	1.470	7.167	4.706	4.970	2.208	11.563	6.599	7.097
Midas-AL	0.535	2.670	1.792	1.935	1.366	6.636	4.529	4.781	1.895	9.014	6.290	6.536
Midas-Evt	0.534	2.664	1.793	1.936	1.376	6.725	4.484	4.736	1.895	9.325	6.138	6.449
MidasAs-AL	0.513	2.560	1.758	1.897	**1.312**	**6.368**	4.502	4.752	**1.790**	**8.515**	6.171	6.416
MidasAs-Evt	0.516	2.574	1.755	1.894	1.346	6.624	**4.431**	**4.684**	1.906	9.410	**6.074**	**6.372**

Table A.8 Model Confidence Set - Different Country Group

This table reports the results of the 5% Model Confidence Set (MCS) for different country groups at the 1% and 5% quantile levels for 1-, 5- and 10-day forecast horizons, respectively. The entry in each column presents the number of times out of 43 indices, that the model in row is excluded from the 5% MCS. L_Q denotes the quantile loss function of (20) and L_{FZG} is the FZG loss function of Fissler et al. (2015) given in (21). The range statistic in (22) is used to the equivalence test of the MCS. Lower values corresponds to superior performance.

	1-day horizon				5-day horizon				10-day horizon			
	1%		5%		1%		5%		1%		5%	
	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}
Panel A: Developed Stock Markets												
GARCH-Fhs	1	1	7	6	2	2	0	0	3	4	2	2
GARCH-Evt	2	1	6	6	1	1	0	0	3	4	1	1
GJR-Fhs	0	0	1	1	1	1	0	0	4	5	1	1
GJR-Evt	0	0	1	1	1	1	0	0	4	5	1	1
Sav-AL	1	1	10	10	6	5	2	2	15	16	5	5
Sav-Evt	3	2	11	11	6	6	1	1	8	11	4	4
As-AL	1	1	0	0	5	5	2	2	12	13	4	4
As-Evt	0	0	0	0	3	3	1	1	6	10	1	2
Midas-AL	1	1	9	10	2	2	2	2	6	6	2	2
Midas-Evt	2	2	8	8	4	4	1	1	5	5	0	0
MidasAs-AL	0	0	0	0	0	0	1	1	0	0	0	0
MidasAs-Evt	0	0	1	1	1	1	0	0	2	3	0	0
Panel B: Emerging Stock Markets												
GARCH-Fhs	6	6	8	7	1	1	1	0	0	0	0	0
GARCH-Evt	5	5	7	7	2	2	1	0	0	0	0	0
GJR-Fhs	2	2	1	1	1	1	1	0	2	2	0	0
GJR-Evt	2	2	1	1	1	1	1	0	2	2	0	0
Sav-AL	7	7	9	8	3	3	5	5	4	3	1	2
Sav-Evt	8	7	10	9	3	3	5	4	1	5	1	2
As-AL	4	4	0	0	2	2	3	2	4	4	3	3
As-Evt	4	4	1	1	3	3	1	1	2	6	0	1
Midas-AL	6	6	9	8	2	2	2	1	1	1	3	2
Midas-Evt	8	7	10	9	1	1	0	0	0	3	0	1
MidasAs-AL	0	0	2	2	1	1	0	0	0	0	0	0
MidasAs-Evt	4	4	2	2	1	1	0	0	1	4	0	0

A.5. Robustness Checks: Alternative Estimation Windows

Table A.9 Out-of-Sample Forecast Losses - Alternative Estimation Windows

This table provides the average out-of-sample forecast losses for alternative rolling window length at the 1% and 5% quantile levels for 1-, 5- and 10-day horizons, respectively. L_Q denotes the quantile loss function of 20 and L_{FZG} is the FZG loss function of Fissler et al. (2015) given in (21). The L_Q and L_{FZG} values are multiplied by 10^4 and 10^3, respectively, to facilitate presentation. Lower values correspond to superior performance. Bold numbers indicate best methods in each column.

	Panel A: 1-day horizon				Panel B: 5-day horizon				Panel C: 10-day horizon			
	1%		5%		1%		5%		1%		5%	
Models	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}
Panel A: 1500-observtion Estimation Window												
GARCH-Fhs	0.482	2.406	1.646	1.772	1.253	6.089	4.090	4.297	1.686	8.011	5.721	5.894
GARCH-Evt	0.481	2.400	1.645	1.772	1.248	6.063	4.080	4.286	1.674	7.959	**5.706**	**5.879**
GJR-Fhs	0.512	2.555	1.712	1.844	1.328	6.446	4.242	4.459	1.844	8.772	5.888	6.076
GJR-Evt	0.474	2.364	1.630	1.754	**1.229**	**5.967**	4.071	4.276	1.712	8.127	5.761	5.935
Midas-AL	0.481	2.400	1.647	1.774	1.267	6.146	4.051	4.255	1.702	8.103	5.771	5.950
Midas-Evt	0.498	2.482	1.671	1.800	1.329	6.482	4.232	4.456	1.903	9.578	6.027	6.367
MidasAs-AL	**0.467**	**2.329**	**1.616**	**1.740**	1.245	6.040	**4.022**	**4.225**	**1.664**	**7.909**	5.721	5.887
MidasAs-Evt	0.483	2.411	1.642	1.767	1.351	6.640	4.262	4.499	1.995	10.187	6.077	6.437
Panel B: 2000-observation Estimation Window												
GARCH-Fhs	0.482	2.405	1.642	1.770	1.255	6.107	4.056	4.271	1.658	7.899	**5.622**	**5.811**
GARCH-Evt	0.480	2.396	1.642	1.770	1.254	6.099	4.048	4.262	1.680	7.999	5.629	5.820
GJR-Fhs	0.508	2.536	1.702	1.835	1.321	6.422	4.206	4.431	1.820	8.668	5.813	6.017
GJR-Evt	0.474	2.364	1.626	1.752	1.237	**6.012**	4.051	4.265	1.695	8.065	5.682	5.874
Sav-AL	0.484	2.401	1.888	2.039	1.404	6.805	4.453	4.697	2.003	9.555	6.011	6.230
Sav-Evt	0.481	2.398	1.628	1.755	1.348	6.544	4.177	4.399	1.933	9.809	5.754	6.087
As-AL	0.472	2.342	1.896	2.049	1.283	6.221	4.512	4.763	1.894	9.037	6.020	6.242
As-Evt	0.468	2.334	**1.598**	**1.722**	1.275	6.205	4.055	4.271	1.855	9.614	5.623	6.067
Midas-AL	0.481	2.399	1.646	1.775	1.276	6.197	4.035	4.249	1.672	7.980	5.720	5.916
Midas-Evt	0.499	2.488	1.671	1.802	1.336	6.525	4.222	4.456	1.884	9.528	5.960	6.327
MidasAs-AL	**0.467**	**2.331**	1.616	1.741	**1.251**	6.076	**4.009**	**4.221**	**1.646**	**7.848**	5.676	5.868
MidasAs-Evt	0.485	2.418	1.641	1.769	1.360	6.697	4.260	4.510	1.987	10.201	6.048	6.440

Table A.10 Model Confidence Set - Alternative Estimation Windows

This table reports the results of the 5% Model Confidence Set (MCS) for alternative lengths of rolling window at the 1% and 5% quantile levels for 1-, 5- and 10-day forecast horizons, respectively. The entry in each column presents the number of times out of 43 indices, that the model in row is excluded from the 5% MCS. L_Q denotes the quantile loss function of (20) and L_{FZG} is the FZG loss function of Fissler et al. (2015) given in (21). The range statistic in (22) is used to the equivalence test of the MCS. Lower values corresponds to superior performance.

	1-day horizon				5-day horizon				10-day horizon			
	1%		5%		1%		5%		1%		5%	
	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}
Panel A: 1500-observtion Estimation Window												
GARCH-Fhs	10	10	17	18	0	0	0	0	3	4	1	1
GARCH-Evt	8	7	15	15	1	1	1	1	3	4	0	0
GJR-Fhs	17	17	27	27	3	3	8	8	8	9	0	1
GJR-Evt	5	5	7	7	0	0	0	0	8	9	1	1
Midas-AL	8	8	17	17	2	2	0	0	5	6	0	0
Midas-Evt	28	27	32	33	11	13	17	17	13	21	10	15
MidasAs-AL	0	0	0	0	0	0	0	0	1	1	2	1
MidasAs-Evt	13	12	20	20	14	16	20	20	12	28	9	17
Panel B: 2000-observation Estimation Window												
GARCH-Fhs	6	4	28	27	2	2	3	1	2	2	0	0
GARCH-Evt	7	7	28	29	1	1	2	1	3	2	0	0
GJR-Fhs	18	17	30	30	4	4	9	9	5	4	2	2
GJR-Evt	2	2	19	19	1	1	1	3	2	2	1	1
Sav-AL	8	7	43	43	8	9	13	15	17	15	2	4
Sav-Evt	9	9	17	17	5	5	8	8	9	17	2	2
As-AL	3	2	43	43	1	1	14	14	9	7	4	4
As-Evt	2	2	5	5	1	1	1	1	7	16	3	2
Midas-AL	5	5	30	30	1	2	0	0	2	2	1	1
Midas-Evt	25	24	40	40	9	13	14	16	12	19	10	13
MidasAs-AL	0	0	11	10	0	0	1	1	1	1	0	0
MidasAs-Evt	9	10	30	30	12	14	19	21	12	26	10	14

Table A.11 Model Confidence Set Results - 10% MCS

This table reports the results of the 10% Model Confidence Set (MCS) at the 1% and 5% quantile levels for 1-, 5- and 10-day forecast horizons, respectively. The entry in each column presents the number of times out of 43 indices, that the model in row is excluded from the 5% MCS. L_Q denotes the quantile loss function of (20) and L_{FZG} is the FZG loss function of Fissler et al. (2015) given in (21). The range statistic in (22) is used to the equivalence test of the MCS. Lower values corresponds to superior performance.

	1-day horizon				5-day horizon				10-day horizon			
	1%		5%		1%		5%		1%		5%	
Models	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}
GARCH-Fhs	11	11	17	18	5	5	1	0	4	5	2	3
GARCH-Evt	11	11	17	18	4	4	1	0	4	4	1	2
GJR-Fhs	4	4	3	4	2	2	1	0	9	8	2	3
GJR-Evt	4	4	3	4	2	2	1	0	10	9	3	3
Sav-AL	13	13	26	27	13	12	12	10	28	27	12	12
Sav-Evt	14	14	27	29	11	10	11	10	20	26	7	13
As-AL	10	9	2	2	10	11	9	7	27	27	13	15
As-Evt	6	6	2	2	7	7	7	6	18	28	5	10
Midas-AL	13	13	24	25	4	4	5	3	10	11	3	4
Midas-Evt	13	14	23	23	6	6	1	1	9	12	1	2
MidasAs-AL	2	1	4	4	1	1	2	1	0	0	0	0
MidasAs-Evt	7	7	7	7	3	3	0	0	7	8	0	1

Table A.12 Model Confidence Set - Alternative Elimination Rule

This table reports the results of the 5% Model Confidence Set (MCS) at the 1% and 5% quantile levels for 1-, 5- and 10-day forecast horizons, respectively. The entry in each column presents the number of times out of 43 indices, that the model in row is excluded from the 5% MCS. L_Q denotes the quantile loss function of (20) and L_{FZG} is the FZG loss function of Fissler et al. (2015) given in (21). The using the semi-quadratic statistic in Hansen et al. (2011) is used to the equivalence test of the MCS. Lower values corresponds to superior performance.

	Panel A: 1-day				Panel B: 5-day				Panel C: 10-day			
	1%		5%		1%		5%		1%		5%	
Models	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}	L_Q	L_{FZG}
GARCH-Fhs	6	6	3	3	1	1	0	0	2	2	1	1
GARCH-Evt	5	5	4	4	1	1	0	0	2	2	1	1
GJR-Fhs	1	1	1	1	1	1	0	0	2	3	1	1
GJR-Evt	3	3	1	1	1	1	0	0	2	3	1	1
Sav-AL	9	8	14	15	3	3	4	4	8	10	5	5
Sav-Evt	9	8	13	13	3	3	2	3	3	8	1	4
As-AL	3	3	1	1	2	3	2	2	11	11	3	4
As-Evt	3	3	0	0	2	2	2	0	3	11	1	4
Midas-AL	6	6	14	15	1	1	0	0	2	3	0	0
Midas-Evt	7	7	14	14	1	1	0	0	1	3	0	0
MidasAs-AL	1	1	0	0	0	0	0	0	0	0	0	0
MidasAs-Evt	3	3	0	0	1	1	0	0	1	3	0	0

Table A.13 Model Confidence Set - Alternative Bootstrapping Methods

This table reports the results of the 5% Model Confidence Set (MCS) at the 1% and 5% quantile levels for 1-, 5- and 10-day forecast horizons, respectively. The entry in each column presents the number of times out of 43 indices, that the model in row is excluded from the 5% MCS. L_Q denotes the quantile loss function of (20) and L_{FZG} is the FZG loss function of Fissler et al. (2015) given in (21). The range statistic in (22) is used to the equivalence test of the MCS. The test statistic is constructed using alternative bootstrapping methods. Lower values corresponds to superior performance.

	1-day horizon				5-day horizon				10-day horizon			
	1%		5%		1%		5%		1%		5%	
	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}	L_Q	L_{QS}
Panel A: Use of stationary bootstrapping												
GARCH-Fhs	7	6	3	3	0	0	0	0	2	3	1	1
GARCH-Evt	5	5	4	4	0	0	0	0	2	3	1	1
GJR-Fhs	2	2	1	1	1	1	0	0	3	4	1	1
GJR-Evt	3	3	1	1	1	1	0	0	3	4	1	1
Sav-AL	10	10	16	19	6	6	5	5	10	13	5	8
Sav-Evt	8	8	13	14	5	5	3	3	3	11	3	5
As-AL	3	3	1	1	3	3	4	4	10	10	4	5
As-Evt	3	3	0	0	1	1	2	2	4	13	1	6
Midas-AL	6	7	15	15	0	0	1	1	3	4	0	0
Midas-Evt	8	8	14	15	1	1	0	0	1	5	0	0
MidasAs-AL	1	1	0	0	0	1	0	0	0	0	0	0
MidasAs-Evt	3	3	0	0	0	0	0	0	2	6	0	0
Panel B: Block bootstrapping of length 2												
GARCH-Fhs	6	6	3	3	0	1	0	0	2	2	1	1
GARCH-Evt	5	5	4	4	0	1	0	0	2	2	1	1
GJR-Fhs	1	1	1	1	1	1	0	0	3	3	1	1
GJR-Evt	3	3	1	1	1	1	0	0	3	3	1	1
Sav-AL	9	10	18	18	5	3	5	4	11	12	6	4
Sav-Evt	8	8	13	13	3	3	2	3	3	7	3	7
As-AL	3	3	1	1	2	3	3	3	11	10	4	5
As-Evt	3	3	0	0	1	2	2	2	5	12	1	6
Midas-AL	5	6	16	16	0	1	1	1	3	3	0	0
Midas-Evt	8	8	14	15	1	1	0	0	1	4	0	0
MidasAs-AL	1	1	0	0	0	0	0	0	0	0	0	0
MidasAs-Evt	3	3	0	0	0	1	0	0	2	4	0	0
Panel C: Block bootstrapping of length 6												
GARCH-Fhs	6	6	3	3	1	0	0	0	2	2	1	1
GARCH-Evt	5	5	4	4	1	0	0	0	2	2	1	1
GJR-Fhs	1	1	1	1	2	1	0	0	2	2	1	1
GJR-Evt	3	3	1	1	2	1	0	0	2	2	1	1
Sav-AL	7	7	14	14	3	3	4	3	11	11	4	5
Sav-Evt	8	7	13	14	3	3	1	1	4	9	1	4
As-AL	3	3	1	1	2	1	1	1	10	11	3	3
As-Evt	3	3	0	0	1	1	0	0	3	11	1	4
Midas-AL	6	6	12	13	1	0	0	0	2	2	0	0
Midas-Evt	7	6	12	12	2	1	0	0	1	3	0	0
MidasAs-AL	1	1	0	0	0	0	0	0	0	0	0	0
MidasAs-Evt	3	3	0	0	1	0	0	0	1	3	0	0