The Richest Wins Them All: Triggering the Benevolent Provision of Public Goods by Federal Transfers ☆

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Abstract

This paper provides a theoretical foundation for the use of commonly used federal transfers when multi-layered public good policies coexist. To ensure consent of differently rich states we constrain the federal government to attain Pareto superior allocations relative to the decentralized policy scenario. We show when a uniform federal price combined with commonly used transfer criteria (equality, decentralized emission level shares, juste retour) deliver Pareto superior allocations. We find that transfer criteria based on equality or decentralized shares can be effective, but the states' reactions to the federal transfer hamper the federal policy intervention. In the absence of interstate transfers (juste retour), we find that Pareto superior allocations are not attainable. We identify an endogenously emerging federal minimum price which ensures consent of all states. At this minimum price the richest state agrees to carry a disproportionately large share of the federal policy cost and becomes the benevolent hegemon.

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1. Introduction

Policy-making on multiple governmental layers often coexists while or precisely because the layers pursue different interests. In federal and quasi federal regimes the authority of upper governmental layers is often limited and their decision-making requires the lower layers' consent. If the provision of a public good shall be improved by a federal government the states' consent becomes arguably a crucial element. States' consent is not only influenced by a better public good provision. At the heart of the states' consent often stand federal fiscal withdrawals from and injections into the states' economies, summarized under the term "federal transfers". In reality, these transfers and transfer negotiations are often encountered. Likely, as a reply by policy pragmatism to a complex world, the transfer criteria often follow simple rules of thumb as a mixture derived from welfare economics, moral considerations, and state's self-interest.[1]

We aim at providing a theoretical foundation to guide the use of commonly used federal transfers. We consider differently wealthy states embedded in a multi-layered policy regime in which federal policy must have the states' consent, a subject that has received little attention in the previous literature. While states and federal policies simultaneously coexist, states' consent is ensured by constraining the federal government to attain Pareto superior allocations compared to the decentralized scenario. Within the context of the model developed and the federal transfer criteria considered we determine when a uniform federal price delivers Pareto superior allocations and when it fails. In modeling a uniform federal price we seek to pay tribute to current developments not only but also on the background of carbon pricing (Edenhofer et al., 2017; Cramton et al., 2015).

Using a general equilibrium framework we model coexisting, strategic multi-

[1]For empirical relevance of the simple transfer criteria see e.g. Warleigh (2004); FOEN (2016); EC (2015); Carbon Pricing Leadership (2016).

national and multi-layer policies for providing a global public good. The global public good has the characteristic of mitigating a transboundary public bad created by an input used in production. Therefore, the production side is regulated by state and federal price policies. The federal price revenues are redistributed as federal transfers to consumers. We study three types of transfer criteria: juste retour, equality, and decentralized public bad creating input shares (referred to as decentralized criterion).

The model developed considers the federal government to be a Stackelberg leader which takes into account how its federal policy price influences the states' policy prices. Whereas state governments act as Nash players with regard to state and federal prices. Besides the impact of the marginal utility of consumption on policy and the role of transfers, as determined in previous literature, we also find that federal transfers and their anticipation by state governments have an important policy impact. If each state government takes into account the impact of its policy on federal transfers, this greatly influences the state's policy which in turn hampers the ability of the federal government to achieve Pareto superior allocations.

The transfer criteria create an institutional tipping point as they influence whether the federal regime is functionable— in delivering Pareto improvements— or not. We find, inter alia, that a federal government can improve on strategic states' policies if the federal government uses a uniform price and transfers based on equality or decentralized criteria. We demonstrate that the utility of the richest state is maximized at the lower range of the set of federal prices while all other utilities are maximized at higher federal price levels. The richest state accepts the federal policy and accepts to bear a large share of the cost of the federal policy if the price chosen is in the neighborhood of the price that maximizes its utility— similarly, as if the federal government assigns a weight of one to the richest state in the context of a social welfare function. As the result of such federal policy the richest state becomes a benevolent hegemon of the federal regime.[2]

In addition to state policies, the federal price further internalizes the transboundary externality, which benefits all states, while the federal transfer either increases the income of relatively poor states (equality criterion) or rewards higher decentralized income levels (decentralized criterion). We find that while the applicability of the equality criterion is always limited by the heterogeneity of the states' capital endowments, the transfer based on the decentralized criterion does not always face such a restriction.

In contrast, the juste retour criterion does not generate Pareto improvements. This result differs to that of d'Autumne et al. (2016); Sandmo (2004); Shiell (2003) who assume that the juste retour transfer is lump-sum from the state governments' perspective. With a juste retour criterion states know exactly to what does the federal transfer amounts to. Therefore, the most reasonable assumption is that the federal transfer must not be taken as lump-sum by the state governments. The juste retour criterion corresponds to the case in which transfers across states do not occur since states receive as federal transfer exactly what they paid to the federal government. Interestingly, in Chichilnisky and Heal (1994) and Sandmo (2007) Pareto optimality is not achievable in the absence of interstate transfers. Here we find that not even Pareto improvements are attainable in the absence of interstate transfers.

Previous authors examine policies, transfers and state-federal regulation with an unconstrained federal or central government (Chichilnisky and Heal, 1994; Sandmo, 2007; Helm, 2003; Williams, 2012; Köthenbürger, 2002). By "unconstrained" , we mean that the federal or central government's policy intervention does not necessarily require the states' consent in terms of ensuring

[2] The term hegemon either signals a benevolent or a coercive power depending on whether it is embedded in the neo-liberal or neo-realist version of hegemonic stability theory. The neo-liberal version characterizes the hegemon as benevolent because the hegemon bears a disproportionate share of the costs of providing public goods that benefit all (Yarbrough, 2001).

Pareto improvements relative to the decentralized solution. If Pareto improvements are not required, for instance, Helm (2003) finds that central regulation underperforms decentralized regulation. A key element of Helm (2003)s result is the transfers ex-ante determination by negotiations between states. A similar finding, but in a political economy context, is obtained by Luelfesmann et al. (2015). While we keep the idea of the self-interest of states, which drives Helm and Luelfesmann et al. as a pivotal element, we depart from these papers by allowing the simultaneous coexistence of state and federal policies in the fashion of Williams (2012). Chichilnisky and Heal (2000) consider a constrained central regulator, do not include a multi-layered policy system, and arrive at similar findings as in their previous work. Böhringer and his colleagues consider fiscal state and federal interactions in the context of climate change mitigation by means of a CGE model for the Canadian economy (Böhringer et al., 2016). They identify vertical fiscal externalities as a major determinant of the welfare changes triggered by a state's climate policy.

One implication of our results adds to the minimum price debate, lively discussed for instance in the EU ETS, an argument solely based on Pareto improvements ensuring states' consent whereas previous debates focus on its benefits by reducing price uncertainty (Abrell and Rausch, 2016; Philibert, 2009). To our knowledge, we add to the existing literature a new explanation in terms of the emergence of a benevolent hegemon and a new argument in favor of minimum prices for global public goods. We show that an existing federal regime structure can give rise to a benevolent hegemon, if the federal transfers are set wisely. Our interpretation turns the theory of Olson (Olson, 1965, 1986) upside down. While Olson demonstrates that a benevolent hegemon is often willing to create a multinational regime, we demonstrate how a multinational regime can create a benevolent hegemon.

In the remainder, we use for the sake of simplicity of our arguments, the word "emissions" as a synonymous for the source of a global or federal externality (a public bad) and "emission mitigation" as a synonymous for the provision of a global or federal public good.

2. The model

We consider a federal system comprised of n member states ($i = 1, ..., n$). Across member states an identical final good is produced using capital and emissions. In each state there is a representative household and population is normalized to one. Households derive utility from consuming the final good, but the federation's aggregate emissions negatively affect their well-being. The households own capital which they rent out to final good producers, but capital is immobile across states. In each state there is a government that sets an emission price on the local firm to regulate local emissions. In turn, state i's government tranfers emission-pricing revenues to state i's household. In addition there is a federal government which sets a uniform price on the emissions of all firms and redistributes back federal revenues to the households of all states.

The primary focus of our model lies upon the transfer criteria of federal government's revenues. We compare two different institutional settings: First, we derive the lower benchmark, called the decentralized solution. In the decentralized solution, only state governments set emission prices. Second, we deploy a two-layered governmental system by introducing a federal government. State governments act as Nash players by taking as given the emissions price of other states and federal governments. Instead the federal government acts as a Stackelberg-leader for all economic agents and state governments. Coexisting with states' policies, the federal government acts if and only if it can ensure Pareto improvements relative to the decentralized solution. The federal government redistributes its emission-price revenues based on three different transfer criteria: equality, juste retour and transfers based on decentralized emission level shares. We study the impact of the transfer criterion on the ability of the federal government to improve upon the decentralized solution.

2.1. Economic agents

2.1.1. Firms

The firm of state i employs capital k_i and emissions e_i using a constant returns to scale technology to produce final good y_i. Taking prices as given the firm chooses k_i and e_i to maximize profits

$$\max_{k_i, e_i} \left\{ \left(y_i - r_i k_i - \left(\tau_i + T \right) e_i \right) | y_i = A k_i^{\alpha_K} e_i^{\alpha_E} \right\}. \tag{1}$$

The price of the final good is presumed to be numéraire. The parameters $\alpha_K > 0$, $\alpha_E > 0$ are the production elasticities of capital and emissions, respectively, with $\alpha_K + \alpha_E = 1$, and A is an efficiency parameter. The rental rate of capital of state i is denoted by r_i, τ_i is state i's price of emissions, and T is the uniform federal emissions price. Therefore, firm i's unit cost of emissions equals $\tau_i + T$. Firms maximize profits by setting the marginal product of each factor equal to its respective price. The marginal cost (mc_i) of producing good y_i equals

$$mc_i = \frac{r_i^{\alpha_K} \left(\tau_i + T \right)^{\alpha_E}}{\alpha_K^{\alpha_K} \alpha_E^{\alpha_E} A}. \tag{2}$$

Zero profits imply

$$mc_i = 1 \tag{3}$$

Hereafter, the conditional demand for capital k_i can be derived as follows

$$k_i = \frac{\partial mc_i}{\partial r_i} y_i = \frac{\alpha_K}{r_i} mc_i y_i \tag{4}$$

and similarly for e_i. Hence $k_i = \alpha_K y_i / r_i$ and $e_i = \alpha_E y^i / \left(\tau_i + T \right)$.

2.1.2. Households

Each household derives satisfaction from consuming the final good. Aggregate federal emissions, $e = \sum_i e_i$, negatively affect each household's utility. We assume an additively separable utility function. The utility function of the representative household of state i is given by $u^i(c_i, e)$ where c_i denotes consumption of the final good, and $\partial u^i / \partial c_i > 0$, $\partial^2 u^i / \partial c_i^2 \leq 0$, $\partial u^i / \partial e < 0$, and $\partial^2 u^i / \partial e^2 \leq 0$. The latter implies that the marginal utility loss from emissions is the larger, the higher emissions are.

Households receive transfers from state and federal governments as lump-sum income.[3] Taking prices and local and aggregate emissions as given, the

[3]While households take all governmental transfers as given, state governments may have

household of state i chooses the level of consumption c_i that maximizes its utility subject to the budget constraint

$$c_i = r_i \overline{k}_i + \tau_i e_i + \pi_i \tag{5}$$

where $r_i \overline{k}_i$ is the return to capital endowment $\overline{k}_i$ and π_i and $\tau_i e_i$ are, respectively, federal and state-i transfers to the household of state i. Since the household takes emissions as given, the solution to its optimization problem is reduced to setting c_i equal to income.

2.1.3. Market clearing

Capital market clearing in each state implies that capital demand k_i equals household i's capital endowment (i.e. $k_i = \overline{k}_i$). Market clearing in final goods is given by

$$\sum_{i=1}^{n} c_i = \sum_{i=1}^{n} y_i. \tag{6}$$

Let $e = \sum_{i=1}^{n} e_i$ and $\overline{k} = \sum_{i=1}^{n} \overline{k}_i$ denote, respectively, aggregate federal emissions and aggregate federal capital. In what follows, we derive expressions for all variables in terms of τ_i and T, which we then use to solve the optimization problems of state and federal governments. These expressions take into account the first order conditions of households and firms as well as market clearing conditions. Substituting (4) into (3) and solving for r_i we obtain,

$$r_i = R_i(\tau_i, T) = \left(\frac{\alpha_K^{\alpha_K} \alpha_E^{\alpha_E} A}{(\tau_i + T)^{\alpha_E}} \right)^{\frac{1}{\alpha_K}}. \tag{7}$$

r_i is trivially decreasing in $(\tau_i + T)$, reflecting that if τ_i or T increase, the remuneration that firms can make to the owners of capital must decrease. Since $k_i = \alpha_K y_i / r_i$, using (7) and $k_i = \overline{k}_i$ it follows that

$$y_i = Y_i(\tau_i, T) = \left(\frac{\alpha_E^{\alpha_E} A}{(\tau_i + T)^{\alpha_E}} \right)^{\frac{1}{\alpha_K}} \overline{k}_i. \tag{8}$$

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full information regarding federal transfers and hence they may internalize the effect of their policies on federal transfers. We consider these issues in more detail in the next sections.

Since $e_i = (\alpha_E/\alpha_K)\, r_i\overline{k}_i/\,(\tau_i + T)$ and using (7) we obtain,

$$e_i = E_i\,(\tau_i, T) = \left(\frac{\alpha_E A}{\tau_i + T}\right)^{\frac{1}{\alpha_K}} \overline{k}_i. \tag{9}$$

As it should be, output (8) and emissions (9) decrease with the aggregate cost of emissions in state i, given by $\tau_i + T$. Thus consumption decreases as well. The balance between these two opposing forces — the gain from decreasing emissions and losses in consumption — and the choice of τ_i and T are studied in the next sections 2.2 and 2.4, in which state and federal governments choose τ_i and T.

Aggregate emissions e equal

$$e = E\,(\tau, T) = \sum_{i=1}^{n} \left(\frac{\alpha_E A\overline{k}_i^{\alpha_K}}{\tau_i + T}\right)^{\frac{1}{\alpha_K}}. \tag{10}$$

where $\tau = (\tau_1, \tau_2 ... \tau_n)$. The federal transfer to household i equals

$$\pi_i = \Pi_i\,(\tau, T) = s_i T E\,(\tau, T) \tag{11}$$

where s_i is the transfer share of federal revenues that is passed to the household of state i. In section 3 we precisely define the transfer share s_i which depends on the transfer criterion employed. Zero profits imply $y_i - Te_i = r_i k_i + \tau_i e_i$, substituting this into equation (5) state i's consumption equals

$$c_i = C_i\,(\tau, T) = Y_i\,(\tau_i, T) + \Pi_i\,(\tau, T) - TE_i\,(\tau_i, T)\,. \tag{12}$$

Thus, household i's consumption departure from local production $Y_i\,(\tau_i, T)$ is given by the net federal transfer $\Pi_i\,(\tau, T) - TE_i\,(\tau_i, T)$. Equations (7) – (12), defined in terms of τ_i (for $i = 1, ...n$) and T, are known to all governments. The choice of τ_i and T is explained in the subsequent sections.

2.2. State governments

In each state there is a state government that only cares about the well-being of the household living in the respective state. The government of state i chooses the emission price τ_i to maximize household i's utility while taking the federal emission price T and all other states' emission prices $\tau_{j\ \forall j \neq i}$ as given. The government of state i incorporates into its optimization problem the solution of all households' and firms' optimization problems, and market clearing conditions. In other words, state i incorporates equations (7) − (12) into its optimization by substituting $e = E\left(\tau, T\right)$ and $c_i = C_i\left(\tau, T\right)$ from (9) − (12) into $u^i\left(c_i, e\right)$. We rewrite household i's utility in terms of τ and T as follows $U^i\left(\tau, T\right) \equiv u^i\left(C_i\left(\tau, T\right), E\left(\tau, T\right)\right)$. State i government's problem is given by

$$\max_{\tau_i} u^i\left(C_i\left(\tau, T\right), E\left(\tau, T\right)\right)\Big|_{\tau_j\ \forall j \neq i \text{ and } T}. \tag{13}$$

The first order condition that solves problem (13) is given by

$$U^i_{\tau_i} = \frac{\partial u^i}{\partial c_i}\frac{\partial C_i}{\partial \tau_i} + \frac{\partial u^i}{\partial e}\frac{\partial E}{\partial \tau_i}\Bigg|_{\tau_j\ \forall j \neq i \text{ and } T} = 0. \tag{14}$$

After some algebraic manipulations we get

$$U^i_{\tau_i} = \frac{\partial u^i}{\partial c_i}\frac{\partial E_i}{\partial \tau_i}\tau_i + \frac{\partial u^i}{\partial c_i}\frac{\partial \Pi_i}{\partial \tau_i} + \frac{\partial u^i}{\partial e}\frac{\partial E_i}{\partial \tau_i}\Bigg|_{\tau_j\ \forall j \neq i \text{ and } T} = 0. \tag{15}$$

Since $\partial E_i / \partial \tau_i < 0$ and $r_i \overline{k}_i + \tau_i E_i = Y_i - T E_i$, the first term in equation (15), reflects how the marginal utility of consumption declines due to the impact of τ_i on income absent from the federal transfer $\left(r_i \overline{k}_i + \tau_i E_i\right)$. An increase in τ_i first generates a decline in income from capital returns and the state transfer, which in turn leads to a decrease in household i's consumption. The next term in equation (15) indicates how the marginal utility of consumption is influenced by the impact of τ_i on the federal transfer. If s_i in $\Pi_i = s_i T E$ is constant, for example an s_i that equally distributes the federal revenues to households, then τ_i has a negative impact on Π_i via its effect on state i's emissions e_i. We later differentiate about whether or not the federal transfer Π_i is lump-sum from the state i government's perspective. The last term in equation (15) reflects the marginal utility from emissions reduction due to an increase in τ_i. After deriving and substituting for $\partial E_i / \partial \tau_i$, and rearranging (15) the states' first order conditions implicitly define the states' prices depending solely on the federal emission price. We denote this relation by $t_i(T)$ and get

$$\tau_i = t_i(T) = -\frac{\frac{\partial u^i}{\partial e}}{\frac{\partial u^i}{\partial c_i}} - \frac{\frac{\partial \Pi_i}{\partial \tau_i}}{\frac{\partial E_i}{\partial \tau_i}}\Bigg|_{\tau_j\ \forall j\neq i \text{ and } T} \quad \text{for all } i. \tag{16}$$

Claim 1. *The per-unit cost of emissions from state i's policy* (τ_i) *equals the marginal rate of substitution (MRS) between aggregate emissions reduction and consumption in state i* $\left(-\frac{\partial u^i}{\partial e}/\frac{\partial u^i}{\partial c_i}\right)$ *minus the ratio of partial derivatives of the federal transfer to state i and state i's emissions with regard to* τ_i.

Focusing on the first term on the RHS of equation (16) we observe how state policies can differ across states. Ceteris paribus, a larger marginal dis-utility from aggregate emissions leads to a larger τ_i, whereas a larger marginal utility from consumption leads to a lower τ_i. This effect is equal to Chichilnisky and Heal's (1994) finding in the case of social optima. The impact of the marginal utility of consumption on the provision of public goods has been studied in previous literature (e.g. Chichilnisky and Heal's (1994)). However the impact of federal transfer criteria on state policy choice under the requirement of states' consensus has largely been neglected. The second term $\left(-\frac{\partial \Pi_i}{\partial \tau_i}/\frac{\partial E_i}{\partial \tau_i}\right)$ takes into account how τ_i influences the federal transfer to household i against the decline of emissions in state i due to an increase on τ_i.

If the federal transfer is not lump-sum to state i's government and if the sign of $\partial \Pi_i/\partial \tau_i$ is negative, which we show to hold, the state's emission price τ_i falls below the marginal rate of substitution between e and c_i, the first term of the RHS. This demonstrates that federal transfers can have an important effect of state policy choice.

Claim 2. *If the federal transfer to household i equals* $\pi_i = s_i T E$ *where* s_i *is a positive constant, then*

$$\tau_i = t_i(T) = -\frac{\frac{\partial u^i}{\partial e}}{\frac{\partial u^i}{\partial c_i}} - s_i T\Bigg|_{\tau_j\ \forall j\neq i \text{ and } T} \quad \text{for all } i. \tag{17}$$

2.3. Decentralized equilibrium

Definition A decentralized equilibrium is the quantities $\widetilde{c}_i$, $\widetilde{y}_i$, $\widetilde{k}_i$, $\widetilde{e}_i$ and prices $\widetilde{r}_i$, $\widetilde{\tau}_i$, for all i, such that $\widetilde{c}_i$ solves the optimization problem of household i; $\widetilde{y}_i$, $\widetilde{k}_i$

and $\widetilde{e}_i$ solve the problem of firm i; $\widetilde{\tau}_i$ solves the problem of the state government i; the capital market clearing condition and the market clearing conditions in final goods (6) hold; and $T = 0$.

A tilde over a variable indicates the variable's levels in a decentralized solution. Setting $T = 0$ and $\pi_i = 0$ in equations $(7) - (10)$ the state government's first order condition (15) reduces to

$$\widetilde{\tau}_i = -\left.\frac{\frac{\partial u^i}{\partial e}}{\frac{\partial u^i}{\partial c_i}}\right|_{T=0} \qquad \text{for all } i. \tag{18}$$

State-i's government internalizes the local externalities from state i's emissions. This result lies below the social optimum since it fails to consider the spillover effect of state i's emissions to the neighboring states (Samuelson rule). The resulting decentralized utility levels are

$$\widetilde{U}^i \equiv U^i(\widetilde{\tau}, 0) \text{ given } \widetilde{\tau}_j \ {}_{\forall j \neq i} \text{ and } T = 0 \tag{19}$$

where $\widetilde{c}_i = \widetilde{y}_i = (\alpha_E^{\alpha_E} A / \widetilde{\tau}_i^{\alpha_E})^{\frac{1}{\alpha_K}} \, \overline{k}_i$, $\widetilde{e}_i = (\alpha_E A / \widetilde{\tau}_i)^{\frac{1}{\alpha_K}} \, \overline{k}_i$, and

$$\widetilde{e} = \sum_{j=1}^{n} \left(\frac{\alpha_E A \overline{k}_j^{\alpha_K}}{\widetilde{\tau}_j} \right)^{\frac{1}{\alpha_K}}. \tag{20}$$

In the decentralized solution the marginal rate of substitution between decreasing e and c_i equals $\widetilde{\tau}_i$ (the per unit cost of emissions) which the firm sets equal to the marginal product of emissions.

2.4. The federal government

We introduce a federal government which knows the solution of the households', firms', and state governments' problems and all market clearing conditions and acts as a Stackelberg-leader. In other words, the federal government considers the effect of T on equations (7)-(12) and (15). Using a uniform price on emissions, T, and federal transfers π_i with $\sum_i \pi_i = Te$ its objective is to attain a Pareto superior allocation to the decentralized solution $\widetilde{U}^i$. Let $t = (t_1(T), t_2(T), ...t_n(T))$ denote the vector of states' chosen prices which, as indicated in section (2.2), solely depend on the federal price T. The federal government's problem is given by

$$\max_T \left\{ u^i \left(C_i \left(t,T\right), E\left(t,T\right)\right) \middle| u^j \left(C_j \left(t,T\right), E\left(t,T\right)\right) \geq \widetilde{U}^j \quad \forall\ j \neq i \right\} \tag{21}$$

Equation (21) indicates that the federal government regulates if and only if it can attain Pareto superior allocations relative to the decentralized solution — as to acknowledge the self-interest of the states[4]. This departs from Helm (2003) whose model allows top-level government policy to perform less efficiently than decentralized state policies and from d'Autumne et al. (2016) whose top-level government can delegate tasks down to state governments independent of the ensurance of Pareto-improvements.

Let $\lambda = (\lambda_1, \lambda_2, ...\lambda_n)$, the Lagrangian function related to problem (21) is given by

$$L\left(T,\lambda\right) = u^i \left(C_i \left(t,T\right), E\left(t,T\right)\right) + \sum_{j\neq i} \lambda_j \left[u^j \left(C_j \left(t,T\right), E\left(t,T\right)\right) - \widetilde{U}^j\right]. \tag{22}$$

The n first order conditions for a maximum are given by[5]

$$\begin{aligned} & -\sum_{j=1}^{n} \lambda_j \frac{\partial u^j}{\partial E} \left[\sum_{h=1}^{n} \frac{\partial E_h}{\partial \tau_h} \frac{dt_h}{dT} + \frac{\partial E}{\partial T}\right] \\ & = \sum_{j=1}^{n} \lambda_j \frac{\partial u^j}{\partial c_j} \left[\underbrace{\frac{\partial M_j^d}{\partial \tau_j}}_{\text{negative}} \frac{dt_j}{dT} + \underbrace{\frac{\partial M_j^d}{\partial T}}_{\text{negative}} + \sum_{h=1}^{n} \frac{\partial \Pi_j}{\partial \tau_h} \frac{dt_h}{dT} + \frac{\partial \Pi_j}{\partial T} \right] \end{aligned} \tag{23}$$

[4]Equivalently, the federal objective can be interpreted as addressing the principle of subsidiarity. If the principle of subsidiarity is applied, the federal government should rather fulfill a supporting than subordinating role towards state governments' policies. The federal level shall execute only those tasks that cannot be performed effectively at the state level (Wincott, 2009).

[5]In the appendix we show that $U^i\left(t\left(T\right),T\right)$ is concave in T.

with $\lambda_i = 1$ and $M_i^d = r_i \overline{k}_i + \tau_i e_i$ and

$$\lambda_j \left[U^j (t, T) - \widetilde{U}^j \right] = 0 \quad \text{for all } j \neq i \ . \tag{24}$$

Equation (23) indicates that the federal government considers direct impacts of T on $u^i (C_i, E)$, and also indirect impacts by considering the impact of T on states' prices $t_h (T)$ (for $h = 1, ..., n$). The first order conditions also indicate that either, the federal government takes into account how aggregate emissions impact all households' utilities and how consumption in each state i influences state i utility. Or, the federal government does this only for some households $j \neq q$ while ensuring that the other households utilities are greater than the decentralized scenario $U^q (t, T) > \widetilde{U}^q$ in such case $\lambda_q = 0$.

If some T satisfies $U^j (t, T) > \widetilde{U}^j$ for all $j \neq i$ this implies that $\lambda_{j \ \forall j \neq i} = 0$. If such case exists[6] that would greatly simplify matters and it would allow us to get further analytical insights. In such a case the federal government first order conditions reduce to

$$-\frac{\frac{\partial u^i}{\partial E}}{\frac{\partial u^i}{\partial c_i}} = \frac{\frac{\partial M_i^d}{\partial \tau_i} \frac{dt_i}{dT} + \frac{\partial M_j^d}{\partial T} + \sum_{h=1}^{n} \frac{\partial \Pi_i}{\partial \tau_h} \frac{dt_h}{dT} + \frac{\partial \Pi_i}{\partial T}}{\sum_{h=1}^{n} \frac{\partial E_h}{\partial \tau_h} \frac{dt_h}{dT} + \frac{\partial E}{\partial T}}. \tag{25}$$

Implying that the federal government would chose to implement a federal price T such that household i's marginal rate of substitution between decreasing emissions and private consumption equals the marginal change in income (including the federal transfer) relative to the marginal change of aggregate emissions due to a marginal increase in T.

Definition A Stackelberg equilibrium with transfer criterion $\widehat{\pi}_i$ is the quantities $\widehat{c}_i$, $\widehat{y}_i$,$\widehat{k}_i$, $\widehat{e}_i$ and prices $\widehat{r}_i$, $\widehat{\tau}_i$, $\widehat{T}$ such that $\widehat{c}_i$ solves the optimization problem of household i; $\widehat{y}_i$, $\widehat{k}_i$ and $\widehat{e}_i$ solve the problem of firm i; $\widehat{\tau}_i$ solves the problem of the state government i; $\widehat{T}$ solves the problem of the federal government; the

[6]We discuss and show the existence of such cases in the sequel of this paper.

market clearing conditions of capital and final goods (6) hold; and the balance of payments condition $\widehat{y}_i + \widehat{\pi}_i - \widehat{T}\widehat{e}_i = \widehat{c}_i$ is satisfied for all i.

We use hats to denote a Stackelberg equilibrium solution. In the next section we more carefully specify the federal transfers considered, and present analytical results under specific household utility functions.

3. The role of different federal fiscal transfer criteria

Before we focus on the transfers, let us explain our federal policy choice. While we consider a uniform federal emission price, it might be argued that differentiated prices could fulfill the same purpose. Indeed, if emissions are regulated in a centralized fashion and optimal transfers are absent, Chichilnisky and Heal (1994) show that differentiated prices need to be deployed to attain Pareto optimality. Our justification for using a uniform price is threefold; it is applied in practice and in theory, it is supposed to counteract federal fragmentation, and it serves to foster states' commitment on the basis of reciprocity(Edenhofer et al., 2017; Cramton et al., 2015).

We stay in the tradition of Chichilnisky and Heal by considering income level differences which we model as differences in capital endowments. While Chichilnisky and Heal focus on Pareto optimality, we impose a multi-layered governmental structure, in which the federal government constraint is to find Pareto superior allocations to the resulting decentralized solution.

3.1. Juste retour criterion

Let us consider a transfer which represents an often claimed fairness criterion from the states' perspective. Juste retour literally means "fair return". In other contexts it has been supported by state governments in federal like systems since the federal withdrawals from a state equal the federal re-injections into its economy. This transfer criterion is often demanded from the EU by the EU Member States (Warleigh (2004)) and implicitly considered in the models of d'Autumne et al. (2016) and Shiell (2003). As Shiell (2003) puts it, a state which feels relatively poor might not be willing to pay transfers to relatively richer states and might put its concern into its negotiation position.

The juste retour criterion transfer criterion implies that there are no inter-state transfers as states receive from the federal government exactly what they paid. We use the JR superscript to denote this criterion. Let π_i^{JR} denote the federal transfer under the juste retour criterion, then

$$\pi_i^{JR} = Te_i.$$

Since the state government knows exactly what Te_i amounts to and how τ_i impacts e_i the only reasonable assumption in this case is to consider that each state government takes into account how its respective τ_i influences this transfer, $\partial \Pi_i / \partial \tau_i \neq 0$ (hence the federal transfer is not taken as lump-sum from the state governments).

Proposition 3. *If the federal fiscal transfer to households equals the sum paid by the state's firm and state governments take into account how their respective price τ_i influences the federal transfer, then the federal government cannot achieve a Pareto superior allocation relative to the decentralized solution.*

Proof. Since the federal transfer π_i^{JR} equals Te_i, then its partial derivative with regard to τ_i equals

$$\frac{\partial \Pi_i}{\partial \tau_i} = T \frac{\partial E_i}{\partial \tau_i} \tag{26}$$

Substituting this result into (16) and rearranging we get

$$\tau_i = -\frac{\frac{\partial u^i}{\partial e}}{\frac{\partial u^i}{\partial c_i}} - T \tag{27}$$

Equation (27) indicates that state government i reacts with a 100 percent counter-movement to the federal price. Notice that the aggregate per unit price of emissions is given by $\tau_i + T = -\frac{\partial u^i}{\partial e} / \frac{\partial u^i}{\partial c_i}$ which is as in the decentralized solution where also the per unit price of emissions equals the marginal rate of substitution between decreasing emissions and consumption. $\square$

Since the federal policy addresses the effect of transboundary emissions one would expect that each household could be made better off by the federal policy.

Instead, the juste retour federal tranfers create a pitfall making the federal price redundant. Interestingly, as in Chichilnisky and Heal (1994) and Sandmo (2007) Pareto optimality cannot be established in the absence of interstate transfers. We find here that not even Pareto improvements are achievable without interstate transfers — despite the presence of a strong federal government (Stackelberg leader).

Claim 4. *Pareto improvements are not achievable without interstate transfers.*

3.2. Equality criterion

In this section we present a transfer based on equality and use the superscript EQ to denote variables related to this type of transfer. All households receive an identical federal fiscal transfer such that $s_i^{EQ} = s_j^{EQ} = \frac{1}{n}$. Since there is a single household in each state, the federal transfer given to each household equals

$$\pi^{EQ} = \frac{1}{n} T e. \tag{28}$$

The equality criterion is, for instance, applied by the Swiss Federal government which equally redistributes revenues from the Swiss CO2 levy back to all Swiss residents(FOEN, 2016).

Let

$$u^i(c_i, e) = c_i - g_i e^{\gamma_i}. \tag{29}$$

g_i and γ_i are constants with $g_i > 0$ and $\gamma_i \geqq 1$. Also let κ_i denote the ratio of the capital endowment of state i's household to the capital endowment of the entire federation, $\kappa_i \equiv \overline{k}_i / \overline{k}$. Also let $\kappa_{N-EQ} \equiv \frac{1}{n} \frac{n+\gamma-\alpha_E}{1+\gamma-\alpha_E}$ and $\kappa_{I-EQ} \equiv \frac{1}{n} \frac{n+\gamma-\alpha_E-1}{1+\gamma-\alpha_E-\frac{1}{n}}$.

Proposition 5. *Let $\overline{k}_1 < ... < \overline{k}_n$, $g_i = g$ and $\gamma_i = \gamma \geq 1$ for all i. If i) the federal fiscal transfer is equal across households; ii) each state government does-not (does) take into account the impact of its policy on the federal transfer; and iii) $\kappa_i < \kappa_{N-EQ}$ (κ_{I-EQ}) for all $i = 1, ..., n$, then the federal government can achieve a Pareto superior allocation relative to the decentralized solution.*

Furthermore, there is a uniform federal minimum price $\widehat{T}^{\min} > 0$ that is in the self-interest of all states to pay. The minimum price $\widehat{T}^{\min}$ maximizes the richest state's utility, $\widehat{T}^{\min} \equiv \arg\max_T U^n(t,T)$.

Proof. See Appendix A and Appendix B.

While we give an intuitive explanation of the proof for the case in which state governments do not take into account how τ_i impacts the federal transfer (they set $\partial \Pi_i / \partial \tau_i = 0$ for all $i = 1, ..., n$), we refer to the appendix for technical details.

Rearranging κ_{N-EQ} we get

$$0 < \left(\overline{k} - \overline{k}_i\right) + \left(\gamma - \alpha_E\right)\left(\overline{k}_{av} - \overline{k}_i\right) \tag{30}$$

where $\overline{k}_{av} = \frac{\overline{k}}{n}$. Since $\gamma - \alpha_E > 0$ equation (30) clearly holds for states with capital endowments that are less than the average capital endowment $\left(\overline{k}_i < \overline{k}_{av}\right)$, however for relatively richer states equation (30) becomes a constraint on the federal government's ability to reach Pareto improvements.

Substituting the assumptions and equation (16) into (10), emissions are implicitly defined in terms of T. Substituting $E(t,T)$ and equations (8), (9) and (12) into equation (29) we get[7]

$$U^i(t,T) = \left(\alpha_K \kappa_i + \frac{\alpha_E}{n}\right) A \overline{k}^{\alpha_K} E^{\alpha_E} - \left[\left(\frac{1}{n} - \kappa_i\right)\gamma + 1\right] g E^{\gamma}. \tag{31}$$

If for some T and some i none of the utility constraints is binding (that is $U^{j \neq i} > \widetilde{U}^{j \neq i}$), then the federal government's first order condition equals

$$\frac{dU^i}{dT} = \left[\alpha_E A \left(\chi_i - \theta_i\right)\left(\frac{\overline{k}}{E}\right)^{\alpha_K} - \chi_i \gamma g E^{\gamma-1}\right]\frac{dE}{dT} = 0, \tag{32}$$

[7] We acknowledge that the assumption of linear consumption may seem odd at a first glance. Combing linear consumption with an emission externality ensures a concave utility function and the existence of interior solutions. Therefore, it allows to reproduce similar features as would be obtained using more traditional utility functions such as $\log(c_i)$ while maintaining analytical traceability. Additionally, we ran numerical simulations with other utility functions. The main findings remain similar.

where $\chi_i = \left(\frac{1}{n} - \kappa_i\right)\gamma + 1$, and $\theta_i = \chi_i - \alpha_K \kappa_i - \frac{\alpha_E}{n}$. Since $\frac{dE}{dT} < 0$, the term in square brackets in equation (32) must equal zero. We then show that as long as $\kappa_i < \kappa_{N-EQ}$, the slope of the function $U^i(t,T)$ (for all i) is positive at $T = 0$. This and the concavity of $U^i(t,T)$ ensure that there exists a positive federal price that maximizes the utility of state i. Note that the size of κ_{N-EQ} depends on the parameter values α_E, γ, and the number of states n. The larger n the smaller κ_{N-EQ} becomes, and the smaller the gap among capital endowments must be. Further rearranging κ_{N-EQ}, we get a reflection of state-i's self-interest with regard to the federal policy,

$$\underbrace{\overline{k}_i + (\gamma - \alpha_E)\,\overline{k}_i}_{\text{self-interested perspective}} \quad < \quad \underbrace{\overline{k} + (\gamma - \alpha_E)\,\overline{k}_{av}}_{\text{federal egalitarian perspective}} \quad . \tag{33}$$

The inequality requirement of equation (33) reflects an institutional tipping point, that determines whether or not the federal policy works. Parameters γ and α_E are, respectively, the elasticity of the marginal dis-utility from emissions[8] and the production elasticity of emissions. The difference $(\gamma - \alpha_E)$ points at the ambivalence of emission reduction. The larger the difference of $(\gamma - \alpha_E)$ the smaller κ_{N-EQ} becomes. The inequality requirement also guaranties that $U^i > \widetilde{U}^i$. State government i is concerned with its own economy (the LHS of equation (33) but it is willing to be governed by a federal policy, if its gains from being part of the federal economy with an egalitarian perspective (the RHS) are above the gains from focusing exclusively on its own economy.

When the gap between the capital endowments of the poorest and the richest states' households is too extreme, then the richest states carry a burden $\left(\widehat{T}\widehat{e}_i - \widehat{\pi}_i^{EQ} > 0\right)$ which is too high compared to their benefits from emissions reduction. The more capital is available in a state, the higher is the output of the firm and the higher is the firm's payment to the federal government.

[8]The elasticity parameter of the externality is derived as follows $\partial\left(g\gamma E^{\gamma-1}\right)/\partial E * E/\left(g\gamma E^{\gamma-1}\right) = \gamma$.

Imposing $\kappa_i < \kappa_{N-EQ}$ and solving equation (32) we obtain

$$\widehat{e}^{i}_{N-EQ} = \left(\frac{\alpha_E A}{g\gamma} \frac{\chi_i - \theta_i}{\chi_i} \overline{k}^{\alpha_K} \right)^{\frac{1}{\gamma - \alpha_E}} \tag{34}$$

and

$$\widehat{T}^{i}_{N-EQ} = \theta_i \left(\frac{\alpha_E A}{\chi_i} \right)^{\frac{\gamma-1}{\gamma-\alpha_E}} \left(\frac{g\gamma\overline{k}^{\gamma-1}}{\chi_i - \theta_i} \right)^{\frac{\alpha_K}{\gamma-\alpha_E}} \tag{35}$$

where $\widehat{e}^{i}_{N-EQ}$ and $\widehat{T}^{i}_{N-EQ}$ are the total emission level and the federal emission price which maximizes the utility of the household of state-i alone.

A question remains: which uniform federal emission price is necessary and sufficient such that the federal policy benefits all states, even those who carry the burden of the federal policy? Since $\kappa_1 < ... < \kappa_n$ the federal emission prices that maximize the utility of state i's household for $i = 1, ..., n$ can be ranked such that

$$0 < \widehat{T}^{n}_{N-EQ} < \widehat{T}^{n-1}_{N-EQ} < ... < \widehat{T}^{1}_{N-EQ}. \tag{36}$$

The price that maximizes the riches state's utility, $\widehat{T}^{n}_{N-EQ}$, represents the lower bound of the range of federal prices which solve the federal government's problem. Since $\kappa_i < \kappa_{N-EQ}$ ensures that the slope of all $U^i(t, T)$ is positive at $T = 0$. The federal price $\widehat{T}^{n}_{N-EQ}$ pushes all utilities above their decentralized levels and suffices as the uniform federal minimum price.

At $\widehat{T}^{n}_{N-EQ}$ all the constraints of the federal government's problem are not binding such that equation (25) holds. Substitute n for i into equation (35) to obtain the closed form solution of the uniform federal minimum price,

$$\widehat{T}^{\min}_{N-EQ} \equiv \widehat{T}^{n}_{N-EQ} = \theta_n \left(\frac{\alpha_E A}{\chi_n} \right)^{\frac{\gamma-1}{\gamma-\alpha_E}} \left(\frac{g\gamma\overline{k}^{\gamma-1}}{\chi_n - \theta_n} \right)^{\frac{\alpha_K}{\gamma-\alpha_E}}. \tag{37}$$

If the minimum price $\widehat{T}^{n}_{N-EQ}$ is chosen, the burden of the federal price is carried by the richer states. The maximal burden lies upon the richest state. Maximizing the richest state's utility guarantees staying below the threshold-to-accept for all states. If the minimum price is set, the richest state can be interpreted as a benevolent hegemon created by the federal regime. For the states for which $\overline{k}_i > \overline{k}_{av}$ they solely benefit from the federal policy due to a decrease in emissions. For the states for which $\overline{k}_i < \overline{k}_{av}$, which we call the poorer states, the benefit from federal policy is twofold. First, the federal emission price decreases the externality which has a positive impact on utility. Second, poorer states are net recipients as the federal policy injects more money into the poorer states' economies than what it withdraws from those economies, while the opposite is true if states' capital endowments are above average.[9]

Since for poorer states $\widehat{\pi}_i^{EQ} > \widehat{T}\widehat{e}_i$, the equality criterion can also be understood as an implicit federal positive bias towards poorer states. Our results implicitly follow the claim of Chichilnisky and Heal that poorer states shall become net recipients while richer states shall become net donors based on efficiency grounds. Our work extends their consideration by appreciating the self-interest of the states.[10]

For the case in which each state government takes into account how its emission price τ_i influence the federal transfer (indicated by subscript I) we find that this decreases the gap among capital endowments up to which all states agree to be governed by the federal policy. In this case the restriction pertaining κ_i is smaller. Using equation (16) under the equality criterion the state policy becomes τ_i^{N-EQ} if state governments do not take into account how their price influences the federal transfer, and becomes τ_i^{I-EQ} for the case in which state governments take this into account, and are respectively given by

$$\tau_i^{N-EQ} = -\frac{\frac{\partial u^i}{\partial e}}{\frac{\partial u^i}{\partial c_i}}, \qquad \tau_i^{I} = -\frac{\frac{\partial u^i}{\partial e}}{\frac{\partial u^i}{\partial c_i}} - \frac{1}{n}T \tag{38}$$

In the case in which the state government takes into account how its price

[9]To prove that poorer states are net recipients suppose, i.e. if $\kappa_i < 1/n$,multiply equation (9) by $n/\overline{k}$, then $ne_i/\overline{k} = (\alpha_E/[A_V(\tau_i+T)])^{\frac{1}{\alpha_K}} n\kappa_i$. Multiply equation (10) with $1/\overline{k}$,then $e/\overline{k} = \sum_{i=1}^n \left(\alpha_E \kappa_i^{\alpha_K}/[A_V(\tau_i+T)]\right)^{\frac{1}{\alpha_K}}$. If there exists a state i for which $\kappa_i < 1/n$, there must be a state $j \neq i$ for which $\kappa_j > 1/n$. Thus, $ne_i/\overline{k} < e/\overline{k}$ and $Te_i < Te/n$. Proceed similarly to prove that richer states are net donors, i.e. $Te_i > Te/n$.

[10]When considering differences in preferences, such that $g_i \neq g_j$, we find that also in that case the federal government is able to attain Pareto improvements. For brevity we omit providing this proof but it is available upon request.

influences the federal transfer τ_i^I is reduced in magnitude precisely by the per unit amount of the federal transfer it receives. This has a detrimental effect on the federal government's ability to achieve Pareto improvements. This is reflected by a stronger restriction given by a smaller κ_{I-EQ} relative to κ_{N-EQ}.

Now, the reflection of the states' self-interest is derived by rearranging κ_{I-EQ},

$$\underbrace{\overline{k}_i + (\gamma - \alpha_E)\,\overline{k}_i}_{\text{self-interested perspective}} < \underbrace{\overline{k} + (\gamma - \alpha_E)\,\overline{k}_{av}}_{\text{federal egalitarian perspective}} - \underbrace{\frac{\overline{k} - \overline{k}_i}{n}}_{\text{information term}}. \tag{39}$$

Except for the term $\frac{\overline{k}-\overline{k}_i}{n}$, equation (39) is equal to case N in equation (33). The term $\frac{\overline{k}-\overline{k}_i}{n}$ measures the average capital endowment of the entire federation without state i.

Now the minimum federal emission price equals

$$\widehat{T}_{I-EQ}^{\min} \equiv \widehat{T}_{I-EQ}^{n} = \frac{n}{n-1}\left(\theta_n + \frac{\kappa_n - 1}{n}\right)\left(\frac{\alpha_E A}{\chi_n - 1/n}\right)^{\frac{\gamma-1}{\gamma-\alpha_E}}\left(\frac{g\gamma\overline{k}^{\gamma-1}}{\chi_n - \theta_n - \frac{\kappa_n}{n}}\right)^{\frac{\alpha_K}{\gamma-\alpha_E}}. \tag{40}$$

3.3. Decentralized emission share criterion

In this section, we set the transfer share equal to the emission shares of the decentralized solution. We now use the marking DC to denote this criterion. If each state receives a transfer based on the ratio of its decentralized to aggregate decentralized emission levels so that $s_i^{DC} = \widetilde{e}_i/\widetilde{e}$, then

$$\pi_i^{DC} = \frac{\widetilde{e}_i}{\widetilde{e}} Te.$$

As we have discussed in section 3.2, firms' total payments for emissions are larger the more capital is available in a state. A federal transfer policy that transfers back more to states that paid more may be considered as more agreeable for richer states when compared to an equitable transfer policy. In practice, for instance, the EU ETS' revenue redistribution largely accounts for the historical emission levels before the EU ETS.

Let the utility of each state be equal to equation (29)

Proposition 6. *Let $\overline{k}_i \neq \overline{k}_{j\neq i}$, $g_i = g$ and $\gamma_i = \gamma \geq 1$ for all i. If i) s_i equals the ratio of each state's decentralized to aggregate decentralized emission levels $(\widetilde{e}_i/\widetilde{e})$ so that $\pi_i = \widetilde{e}_i/\widetilde{e}Te$; and ii) each state government does not incorporate the impact of its policy on the federal transfer, then the federal government can achieve a Pareto superior allocation relative to the decentralized solution. Moreover, there exists a uniform federal minimum price, $\widehat{T}^{\min}_{N-DC} > 0$, which is in the self interest of all states to pay. The minimum price maximizes the richest state's utility, $\widehat{T}^{\min}_{N-DC} \equiv \arg\max_T U^n(t,T)$.*

Proof. See Appendix C.

The uniform federal minimum price is now given by

$$\widehat{T}^{\min}_{N-DC} \equiv \widehat{T}^{n}_{N-DC} = (1-\kappa_n)\left[(\alpha_E A)^{\gamma-1}\left(\frac{g\gamma\overline{k}^{\gamma-1}}{\kappa_n}\right)^{\alpha_K}\right]^{\frac{1}{\gamma-\alpha_E}}. \qquad (41)$$

When revenues are redistributed according to decentralized emission shares levels, we arrive at similar findings as with the equality based transfers except of one crucial constraint: there is no threshold for the capital endowment gap as long as the state governments do not take into account how their policies impact the federal transfer. This transfer criterion ensures agreeability across the states who carry the burden of the federal policy. The federal price addresses emission externalities, while when using this transfer the federal government acknowledges the richer states' higher production levels.

Note that $s_i^{DC} = \widetilde{e}_i/\widetilde{e}$ reduces to $s_i^{DC} = \kappa_i = \overline{k}_i/\overline{k}$ such that $\pi_i^{DC} = \kappa_i Te$. Under the decentralized emission levels share criterion the state policy is either τ_i^{N-DC} or τ_i^{I-DC}, depending on whether each state government does not (N) or does (I) take into account how their price influences the federal transfer, using equation (16) we obtain

$$\tau_i^{N-DC} = -\frac{\frac{\partial u^i}{\partial e}}{\frac{\partial u^i}{\partial c_i}}, \qquad \text{or} \qquad \tau_i^{I-DC} = -\frac{\frac{\partial u^i}{\partial e}}{\frac{\partial u^i}{\partial c_i}} - \kappa_i T. \qquad (42)$$

The latter term in τ_i^{I-DC} in equation (42) hints at a pitfall of the transfer criteria subject to states' reaction. The richer a state is the larger is the state's policy counter-reaction to the federal policy. The states' capital endowments shares, κ_i, become again a limiting factor for federal policy to be Pareto improving.

For $\gamma = 1$ we present the restriction on capital endowments. Let $\kappa_{I-DC} \equiv \frac{\alpha_K \kappa_i + \alpha_E}{\alpha_K \kappa_i + 1}$.

Proposition 7. *Let $\overline{k}_i \neq \overline{k}_{j \neq i}$, $g_i = g$ and $\gamma_i = \gamma = 1$ for all i. If i) the transfer is based on the ratio of each state's decentralized to aggregate decentralized emission levels $(\widetilde{e}_i/\widetilde{e})$ and ii) each state government incorporates the impact of its policy on the federal transfer, and iii) $\kappa_i < \kappa_{I-DC}$, then the federal government can achieve a Pareto superior allocation relative to the decentralized solution.*

Proof. See Appendix D.

Consider the requirement $\kappa_i < \kappa_{I-DC}$, and recall that $\alpha_E + \alpha_K = 1$ to see how restrictive this transfer criteria is. The larger the production elasticity of emissions is the larger κ_{I-DC}, which allows this criterion to be Pareto improving . This tells us, that using the decentralized criterion is not a promising transfer if states take into account how their policy influences the federal transfer. Unlike the equity criterion the number of states is irrelevant in the decentralized criterion for the cases we analyze.

3.4. Federal solution space

In addition to the uniform federal minimum price which defines the lower bound of the solution space of possible federal prices, we now specify the range of all admissible federal prices. As depicted in figure 1, all states benefit from the uniform federal price as long as the federal policy intervention pushes each utility level above its decentralized solution. Let $T^i_{\text{ind}} > 0$ denote the federal emission price at which state i's household utility level is equal to that of the decentralized scenario, that is $U^i\left(t, T^i_{\text{ind}}\right) = U^i\left(\tilde{\tau}, 0\right)$. As stated before the smallest federal price is $\widehat{T}^{\min} = \widehat{T}^n$. The highest federal price is the smallest of either the price that maximizes the utility of the poorest state's household or T^i_{ind} for $i = 1, ..., n$.

Corollary 8. *The federal government's solution space is the interval of uniform federal prices that satisfy*

$$T \in \left[\widehat{T}^{\min}, \min\left\{\widehat{T}^{1}, T^{1}_{ind}, ..., \ T^{n}_{ind}\right\}\right].$$

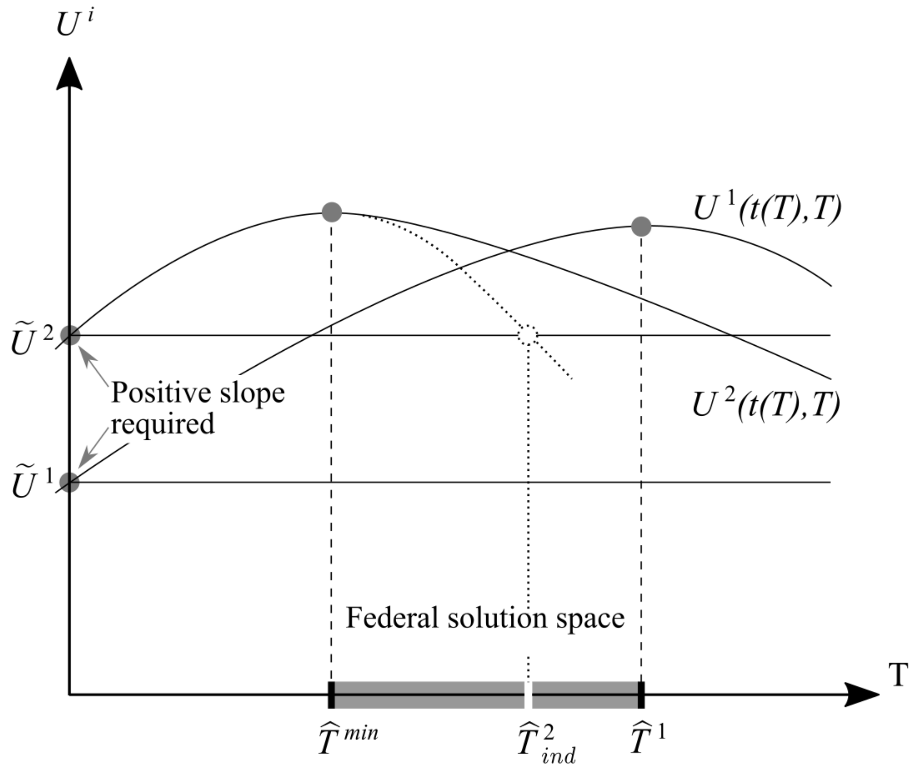

Figure 1: Stylized illustration for $n = 2$ and $\overline{k}_1 < \overline{k}_2$. If the federal price T leads to Pareto improvements, relative to the decentralized scenario $\widetilde{U}^i$, the federal policy solution space is given by the set $\left[\widehat{T}^{\min}, \min\left\{\widehat{T}^{1}, T^{2}_{\text{ind}}\right\}\right]$. The largest admissible T equals $\widehat{T}^1$, if $\widehat{T}^1 < T^2_{ind}$. Instead, if $T^2_{ind} < \widehat{T}^1$ then the largest admissible T is T^2_{ind} (dotted line).

4. Conclusion

We study an entry point for global public good provision in federal regimes and provide a theoretical foundation to guide the use of rules of thumb for federal transfers when consent of states is required. In reality, simple transfer criteria and transfer negotiations are often encountered but mechanisms to ensure the states' consent are not sufficiently understood. The federal policy we consider consists of a uniform federal price complemented with simple federal transfer criteria which exist coexists with states' policies. In this context we analyze the federal policys Pareto improvement potential relative to the decentralized policy outcome. Since optimal transfers leading to Pareto optimality are difficult to attain as this may be the case for pragmatic, economic or political reasons we focus on sub-optimal equilibria and consider instead commonly used federal transfers. We make the case for emissions' mitigation and focus on the case in which wealth differs across the federation's states.

Three types of transfer criteria which have, in different settings, also received attention in previous literature are considered, namely transfers based on equality, decentralized emission shares, and juste retour criteria. We find that transfers based on equality and decentralized emission shares can lead to Pareto improvements. As our theoretical formulation considers states with different capital endowments we find that there is an institutional tipping point in which the gap among the states' capital endowments becomes a constraint for the federal policy. This tipping point in capital endowments is governed by the elasticity of the marginal dis-utility from federal emission and the production elasticity of emissions and the states' reactions to the federal transfer. The states' reactions hamper the federal policy intervention such that the institutional tipping point is attained earlier, when the transfer is anticipated by each state. Opposite to previous results we argue that in the absence of interstate transfers, that is when the juste retour criterion is applied, Pareto superior allocations are not attainable. Juste retour transfers make the federal policy fully ineffective as they induce the states to react with a 100 percent countermovement to the federal policy.

We identify welfare enhancing uniform federal minimum prices for emissions mitigation if federal transfers based on equality or decentralized emission shares are employed. The minimum prices endogenously emerge and are determined by the state with the highest level of capital endowment. The higher the capital endowment within a state the higher is the federal policy burden imposed on it. When such a minimum price, maximizing the richest state's utility, is established, these two transfers trigger benevolent hegemony by the richest state.

Given the simple nature of the model, a natural extension would be to relax the model's assumptions such as introducing capital mobility or a dynamic setting, and to run numerical simulations for federal or federal-like systems like, for instance, the EU, the US or Canada. Additionally, it can be relevant to suppose that state governments act ex-ante as hegemons or frontrunners, in order to correspond to the discussed leadership of specific states and countries like California and Germany. Spillover effects across regulated and unregulated sectors might also provide an interesting facet.

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Appendix A. Proof of Proposition 5 Case N

Let $s^{EQ} = 1/n$, $g_i = g > 0$, $\gamma_i = \gamma \geq 1$,and $\partial \Pi_i / \partial \tau_i = 0$ for all $i = 1, ..., n$. Let l denote the subset of states with capital endowments that are less or equal to the average capital endowment so that $l = \{i \in \{1, ..., n\} \mid \kappa_i \leq 1/n\}$. Note that the average capital endowment equals $k_{av} \equiv \overline{k}/n$ implying that $k_{av}/\overline{k} = 1/n$. Since $\kappa_i \equiv \overline{k}_i/\overline{k}$, then the *kappa* of a state with an average capital endowment equals $1/n$. Also let h denote the set of countries with capital endowments larger than the average capital endowment $h = \left\{i \in \{1, ..., n\} \mid \kappa_i > \frac{1}{n}\right\}$. Suppose, without loss of generality, that $\overline{k}_1 < \overline{k}_2 < ... < \overline{k}_n$. Substitute the assumptions into equation (16) to obtain

$$t_i\left(T\right) = g\gamma e^{\gamma-1} \quad \text{for } i = 1, ..., n. \tag{A.1}$$

Replace τ_i from equation$(A.1)$ into equation (10),

$$E\left(t, T\right) = \left(\frac{\alpha_E A \overline{k}^{\alpha_K}}{g\gamma e^{\gamma-1} + T}\right)^{\frac{1}{\alpha_K}}. \tag{A.2}$$

Rearranging equation $(A.2)$ e is implicitly defined in terms of T,

$$T = \alpha_E A\left(\frac{\overline{k}}{e}\right)^{\alpha_K} - g\gamma e^{\gamma-1}. \tag{A.3}$$

Substituting equations $(A.1)$ and $(A.3)$ into (8), (9) and (12) y_i, e_i, c_i are implicitly defined in terms of T as follows

$$Y_i\left(t, T\right) = A\left(\frac{E}{\overline{k}}\right)^{\alpha_E} \overline{k}_i, \tag{A.4}$$

$$E_i\left(t, T\right) = \kappa_i E, \tag{A.5}$$

and

$$C_i = y_i + \Pi_i - TE_i = A\left(\frac{E}{\overline{k}}\right)^{\alpha_E} \overline{k}_i + \pi_i - T\kappa_i E. \tag{A.6}$$

Replace equations $(A.3)$, $(A.5)$, and $(A.6)$ in equation (29) [11] to implicitly express U^i, in terms of T,

$$U^i\left(t,T\right)=A\left(\frac{E}{\overline{k}}\right)^{\alpha_E}\overline{k}_i+\left(\frac{1}{n}-\kappa_i\right)TE-gE^{\gamma}.$$

Simplifying yields

$$U^i\left(t,T\right)=\left(\alpha_K\kappa_i+\frac{\alpha_E}{n}\right)A\overline{k}^{\alpha_K}E^{\alpha_E}-\left[\left(\frac{1}{n}-\kappa_i\right)\gamma+1\right]gE^{\gamma}. \tag{A.7}$$

The federal government chooses T to maximize U^i while, at the same time, ensuring that U^j does not fall below the decentralized level $\widetilde{U}^j$ $\forall j$,

$$\max\left\{U^i\left(t,T\right)\middle|U^{j\neq i}\left(t,T\right)\geqq\widetilde{U}^{j\neq i}\ \forall\ j\right\} \tag{A.8}$$

We will now demonstrate that the T that maximizes U^n also implies $U^j>\widetilde{U}^j$ for all j. If for some T and some i all the utility constraints are not binding (that is $U^{j\neq i}>\widetilde{U}^{j\neq i}$), then the federal government's first order conditions equal

$$\frac{dU^i}{dT}=Z_i^{N-EQ}\frac{dE}{dT}=0, \tag{A.9}$$

where

$$Z_i^{N-EQ}=\alpha_E A\left(\chi_i-\theta_i\right)\left(\frac{\overline{k}}{E}\right)^{\alpha_K}-\gamma\chi_i gE^{\gamma-1}, \tag{A.10}$$

and

$$\chi_i=\left(\frac{1}{n}-\kappa_i\right)\gamma+1 \text{ and } \theta_i=\chi_i-\alpha_K\kappa_i-\frac{\alpha_E}{n}. \tag{A.11}$$

Thus, either Z_i^{N-EQ} or $\frac{dE}{dT}$ or both must equal zero. Implicit differentiation of equation $(A.2)$ leads to

$$\frac{dE}{dT}=-\frac{E}{\alpha_K\alpha_E A\left(\frac{\overline{k}}{E}\right)^{\alpha_K}+g\gamma\left(\gamma-1\right)E^{\gamma-1}}. \tag{A.12}$$

By definition α_K, α_E, A, and g are positive, $\gamma\geq 1$, and $E\geq 0$. We rule out the case of $E=0$ and therefore the denominator of the RHS is positive. It

[11] Note that $\widehat{\tau}$ represents the vector of all emission price levels chosen by the state governments. It depends on the federal price T as defined in section 2.4.

follows that $\frac{dE}{dT} < 0$. Therefore, Z_i^{N-EQ} must equal zero. Let $\widehat{T}^i$ denote the T that makes Z_i^{N-EQ} equal to zero, and $\widehat{e}^i$ the related level of E. Then, setting $Z_i^{N-EQ} = 0$ and solving yields

$$\widehat{e}^i = \left(\frac{\alpha_E A}{g\gamma} \frac{\chi_i - \theta_i}{\chi_i} \overline{k}^{\alpha_K} \right)^{\frac{1}{\gamma - \alpha_E}}. \tag{A.13}$$

Substitute $\widehat{e}^i$ in equation $(A.3)$. After some manipulations we obtain

$$\widehat{T}^i = \theta_i \left(\frac{\alpha_E A}{\chi_i} \right)^{\frac{\gamma-1}{\gamma-\alpha_E}} \left(\frac{g\gamma \overline{k}^{\gamma-1}}{\chi_i - \theta_i} \right)^{\frac{\alpha_K}{\gamma-\alpha_E}} \tag{A.14}$$

As the federal government seeks to determine a uniform T, which ensures Pareto improvements for all states, let us examine which T suffices. Considering set l and examining χ_i and θ_i from equation $(A.11)$, we see that

$$\chi_i \geq 1, \;\theta_i > 0 \text{ and } \chi_i - \theta_i > 0 \text{ for } i \in l. \tag{A.15}$$

Together with equation $(A.14)$ follows that $\widehat{T}^i > 0$ for all $i \in l$.

Let us examine the behavior of $U^{i\in l}$ on the interval $[0, \widehat{T}^{i\in l})$ by evaluating the slope of $U^{i\in l}$ at the decentralized level, $T = 0$. We know from equation $(A.12)$ that $\frac{dE}{dT} < 0$. Substitute χ_i, θ_i and $E|_{T=0}$, into Z_i^{N-EQ}. After some manipulations, we obtain

$$Z_i^{N-EQ}|_{T=0} = -\,\theta_i g\gamma E^{\gamma-1}\big|_{T=0} \tag{A.16}$$

Since for $i \in l$, all parameters of equation $(A.16)$ are always positive, we find that $Z_{i\in l}^{N-EQ}\Big|_{T=0} < 0$. As $\frac{dE}{dT} < 0$, it follows from equation $(A.9)$ that $U^{i\in l}$ has a positive slope at $T = 0$. Consequently, if there is a role for the federal government then $\widehat{T}$ must be positive, else a negative $\widehat{T}$ would make states in set l worse than the decentralized solution.

Let us examine what $T > 0$ implies for states in set h. To ensure a Pareto improvement, and hence a role for the federal government, for all $i \in h$ the slope of $U^{i\in h}$ must be increasing at $T = 0$. Since

$$\kappa_i < \frac{1 + \frac{1}{n}(\gamma - \alpha_E)}{\alpha_K + \gamma} \text{ for } i = 1, ..., n. \tag{A.17}$$

this implies $\theta_{i\in h} > 0$ and therefore $Z_{i\in h}^{N-EQ}\Big|_{T=0} < 0$ demonstrating that $dU^{i\in h}/dT\big|_{T=0} > 0$.

We now prove that U^i is decreasing on the interval $(\widehat{T}^i, \infty)$. Let $T^b > \widehat{T}^i$. Evaluate the slope of equation $(A.9)$ at T^b. Since we know that $\frac{dE}{dT} < 0$ it suffices to evaluate $Z_i^{N-EQ}\Big|_{T^b}$. Take equation $(A.13)$ to see that $\left(\widehat{e}^i\right)^{\gamma-\alpha_E} = \frac{\alpha_E A(\chi_i-\theta_i)\overline{k}^{\alpha_K}}{\gamma\chi_i g}$. Since $\frac{dE}{dT} < 0$, then $T^b > \widehat{T}^i$ implies

$$E^{\gamma-\alpha_E}\Big|_{T^b} < \left(\widehat{e}^i\right)^{\gamma-\alpha_E} = \frac{\alpha_E A\left(\chi_i-\theta_i\right)\overline{k}^{\alpha_K}}{\gamma\chi_i g}. \tag{A.18}$$

Rearranging we get

$$0 < \left(\alpha_E A\left(\chi_i-\theta_i\right)\left(\frac{\overline{k}}{E}\right)^{\alpha_K} - \gamma\chi_i g E^{\gamma-1}\right)\Bigg|_{T^b} \tag{A.19}$$

the RHS of equation $(A.19)$ is nothing else than $Z_i^{N-EQ}\Big|_{T^b}$ and hence $Z_i^{N-EQ}\Big|_{T^b} > 0$. This implies that U^i is a concave function with a unique maximum at $\widehat{T}^i > 0$.

We now rank the different $\widehat{T}^i s$ for $i = 1, ..., n$. To do so, we first rank $\widehat{e}^i$ for $i = 1, ...n$. Since the $\widehat{e}^i s$ only differ with regard to κ_i we now take the derivative of $\widehat{e}^i$ from equation $(A.13)$ with respect to κ_i. After several manipulations and substitution of χ_i from equation $(A.11)$ we obtain

$$\frac{\partial\widehat{e}^i}{\partial\kappa_i} = \frac{\alpha_K + \frac{\gamma}{n}}{\gamma-\alpha_E}\left[\frac{A\alpha_E\overline{k}^{\alpha_K}}{g\gamma}\frac{\left(\chi_i-\theta_i\right)^{1-\gamma+\alpha_E}}{\chi_i^{\alpha_K+\gamma}}\right]^{\frac{1}{\gamma-\alpha_E}}. \tag{A.20}$$

The first term of the RHS is always positive. Since $\theta_i > 0$ and $\chi_i - \theta_i > 0$ it follows that $\chi_i > 0$. Hence $\frac{\partial\widehat{e}^i}{\partial\kappa_i} > 0$ for all i. We can, therefore, conclude that the higher κ_i, the higher $\widehat{e}^i$. From equation $(A.12)$, it follows that the higher $\widehat{e}^i$ the lower must be $\widehat{T}^i$. Thus,

$$\kappa_1 < ... < \kappa_n \Rightarrow \widehat{e}^1 < ... < \widehat{e}^n \Rightarrow \widehat{T}^n < ... < \widehat{T}^1. \tag{A.21}$$

implying $U^j\big|_{T=\widehat{T}^n} > \widetilde{U}^j$ for all j. $\square$

Appendix B. Proof of Proposition 5 Case *I*

All else equal as in Appendix A except for the assumption that state governments take into account how their respective τ_i influences the federal transfer

$(\partial \Pi_i / \partial \tau_i \neq 0)$. If not mentioned explicitly, the steps are similar to the previous proof such that we only provide the equations without description.

$$t_i\left(T\right) = g\gamma E^{\gamma-1} - \frac{T}{n} \tag{B.1}$$

$$E\left(t,T\right) = \left(\frac{A\alpha_E \overline{k}^{\alpha_K}}{g\gamma E^{\gamma-1} + \frac{n-1}{n}T}\right)^{\frac{1}{\alpha_K}}. \tag{B.2}$$

$$U^i\left(t,T\right) = \frac{\left(\alpha_K n - 1\right)\kappa_i + \alpha_E}{\left(n-1\right)} A\overline{k}^{\alpha_K} E^{\alpha_E} + \left[\frac{\kappa_i n - 1}{n-1}\gamma - 1\right] gE^{\gamma}. \tag{B.3}$$

If $U^j > \widetilde{U}^j$ for all $j \neq i$ for some T and some i, then, the federal government's first order condition equals

$$\frac{dU^i}{dT} = Z_i^{I-EQ}\frac{dE}{dT} = 0 \tag{B.4}$$

where

$$Z_i^{I-EQ} = \frac{n}{n-1}\left[A\alpha_E\left(\chi_i - \theta_i - \frac{\kappa_i}{n}\right)\left(\frac{\overline{k}}{E}\right)^{\alpha_K} + \left(\frac{1}{n} - \chi_i\right) g\gamma E^{\gamma-1}\right]. \tag{B.5}$$

Thus, either Z_i^{I-EQ} or $\frac{dE}{dT}$ or both must equal zero. Implicit differentiation of equation $(B.2)$ leads to

$$\frac{dE}{dT} = \frac{1-n}{n}\frac{E}{A\alpha_E\alpha_K\left(\frac{\overline{k}}{E}\right)^{\alpha_K} + g\gamma\left(\gamma-1\right)E^{\gamma-1}} < 0. \tag{B.6}$$

Thus, at the optimum Z_i^{I-EQ} must equal zero. Solving $Z_i^{I-E} = 0$ for E yields

$$\widehat{e}^i = \left[\frac{A\alpha_E}{g\gamma}\frac{\chi_i - \theta_i - \frac{\kappa_i}{n}}{\chi_i - \frac{1}{n}}\overline{k}^{\alpha_K}\right]^{\frac{1}{\gamma-\alpha_E}}. \tag{B.7}$$

Substituting equation $(B.7)$ into equation $(A.3)$, leads to

$$\widehat{T}^i = \frac{n}{n-1}\left(\theta_i + \frac{\kappa_i - 1}{n}\right)\left(\frac{A\alpha_E}{\chi_i - \frac{1}{n}}\right)^{\frac{\gamma-1}{\gamma-\alpha_E}}\left[\left(\frac{g\gamma}{\chi_i - \theta_i - \frac{\kappa_i}{n}}\right)^{\alpha_K}\left(\overline{k}^{\alpha_K}\right)^{\gamma-1}\right]^{\frac{1}{\gamma-\alpha_E}}. \tag{B.8}$$

Evaluating Z_i^{I-EQ} at $T=0$ yields

$$Z_i^{I-EQ}\Big|_{T=0} = -\frac{ng\gamma}{n-1}\left(\theta_i - \frac{1-\kappa_i}{n}\right)E^{\gamma-1}\Big|_{T=0} \tag{B.9}$$

For $i \in l$ we see that $\left. Z_i^{I-EQ} \right|_{T=0} < 0$. Same as argued in the previous proof, it must be that $\widehat{T}^i > 0$ for $i \in l$. Since

$$\kappa_i < \frac{1}{n} \frac{n - \alpha_E + \gamma - 1}{1 - \alpha_E + \gamma - \frac{1}{n}} \text{ for } i = 1, ..., n. \tag{B.10}$$

then $\frac{1-\kappa_i}{n} < \theta_i$ and consequently $\left. Z_i^{I-EQ} \right|_{T=0} < 0$ for all i. Take equation $(B.7)$ to see that $\left(\widehat{e}^i\right)^{\gamma-\alpha_E} = \frac{A\alpha_E\left(\chi_i - \theta_i - \frac{\kappa_i}{n}\right)\overline{k}^{\alpha_K}}{g\gamma\left(\chi_i - \frac{1}{n}\right)}$. Since $\frac{dE}{dT} < 0$ and $T^b > \widehat{T}^i$ then

$$\left. E^{\gamma-\alpha_E} \right|_{T^b} < \left(\widehat{e}^i\right)^{\gamma-\alpha_E} = \frac{A\alpha_E\overline{k}^{\alpha_K}\left(\chi_i - \theta_i - \frac{\kappa_i}{n}\right)}{g\gamma\left(\chi_i - \frac{1}{n}\right)} \tag{B.11}$$

rearranging we get

$$0 < \left. \left(A\alpha_E \left(\chi_i - \theta_i - \frac{\kappa_i}{n}\right) \left(\frac{\overline{k}}{E}\right)^{\alpha_K} - g\gamma\left(\chi_i - \frac{1}{n}\right) E^{\gamma-1} \right) \right|_{T^b} \tag{B.12}$$

The RHS of equation $(B.11)$ is nothing else than $\left. (n-1)/nZ_i^{I-EQ} \right|_{T^b}$ and hence $\left. Z_i^{I-EQ} \right|_{T^b} > 0$. Therefore, it follows that U^i is a concave function with a unique maximum at $\widehat{T}^i > 0$. The $\widehat{T}^i$s can be ranked by considering

$$\frac{\partial \widehat{e}^i}{\partial \kappa_i} = \frac{n-1}{n} \frac{\alpha_K + \frac{\gamma-1}{n}}{\gamma - \alpha_E} \left[\frac{A\alpha_E\overline{k}^{\alpha_K}}{g\gamma} \frac{\left(\chi_i - \theta_i - \frac{\kappa_i}{n}\right)^{1-\gamma+\alpha_E}}{\left(\chi_i - \frac{1}{n}\right)^{\alpha_K+\gamma}} \right]^{\frac{1}{\gamma-\alpha_E}}.$$

Same as in the previous proof $\widehat{T}^n < ... < \widehat{T}^1$. □

Appendix C. Proof of Proposition 6

All else equal as in Appendix A except for the assumption that the federal transfer Π_i equals $s_i^{DC}TE = \kappa_i TE$. If not explicitly mentioned the steps are similar to the previous proof such that we only provide the equations without description. Substitute the assumptions into equation (16) to obtain

$$t_i(T) = g\gamma E^{\gamma-1} \text{ for all } i = 1, ..., n, \tag{C.1}$$

$$E(t, T) = \left(\frac{A\alpha_E\overline{k}^{\alpha_K}}{g\gamma E^{\gamma-1} + T} \right)^{\frac{1}{\alpha_K}}, \tag{C.2}$$

and

$$U^i(t,T) = (\alpha_K\kappa_i + \alpha_E\kappa_i)\, A\overline{k}^{\alpha_K} E^{\alpha_E} - ge^{\gamma} = A\kappa_i\overline{k}^{\alpha_K} E^{\alpha_E} - gE^{\gamma}. \tag{C.3}$$

If $U^j > \widetilde{U}^j$ (for all $j \neq i$) for some $\widehat{T}$ and some i, then the federal government's first order condition equals

$$\frac{dU^i}{dT} = \left[A\alpha_E\kappa_i\left(\frac{\overline{k}}{E}\right)^{\alpha_K} - g\gamma E^{\gamma-1}\right]\frac{\partial E}{\partial T} = 0. \tag{C.4}$$

Thus, either the term in parenthesis or $\frac{dE}{dT}$ or both equal zero. Implicit differentiation of equation $(C.2)$ leads to

$$\frac{dE}{dT} = -\frac{E}{A\alpha_E\alpha_K\left(\frac{\overline{k}}{E}\right)^{\alpha_K} + g\gamma(\gamma-1)E^{\gamma-1}} < 0. \tag{C.5}$$

Solve the term in parenthesis of equation (C.4) to obtain

$$\widehat{e}^i = \left(\frac{A\alpha_E\kappa_i}{g\gamma}\overline{k}^{\alpha_K}\right)^{\frac{1}{\gamma-\alpha_E}}. \tag{C.6}$$

Substituting equation $(C.2)$ into $(A.3)$ we get,

$$\widehat{T}^i = (1-\kappa_i)\left[\left(A\alpha_E\overline{k}^{\alpha_K}\right)^{\gamma-1}\left(\frac{g\gamma}{\kappa_i}\right)^{\alpha_K}\right]^{\frac{1}{\gamma-\alpha_E}}. \tag{C.7}$$

Note that all terms in equation $(C.7)$ are positive. Thus, the federal price is always positive.

Consider the interval $T \in [0, \widehat{T}^i)$. Since $\frac{\partial E}{\partial T} < 0$. Take the term in parenthesis of equation (C.4) and substitute $\widetilde{e}$, to see

$$(\kappa_i - 1)(g\gamma)^{\frac{\alpha_K}{\gamma-\alpha_E}}\left(A\alpha_E\overline{k}^{\alpha_K}\right)^{\frac{\gamma-1}{\gamma-\alpha_E}} < 0.$$

Thus, $\left.\frac{dU^i}{dT}\right|_{T=0} > 0$. Use equation $(C.6)$ to see that $\left(\widehat{e}^i\right)^{\gamma-\alpha_E} = \frac{A\alpha_E\kappa_i\overline{k}^{\alpha_K}}{g\gamma}$. Since $\frac{dE}{dT} < 0$ note that $E^{\gamma-\alpha_E}|_{T^b} < \left(\widehat{e}^i\right)^{\gamma-\alpha_E}$ for $T^b > \widehat{T}^i$, thus $E^{\gamma-\alpha_E}|_{T^b} < \left(\widehat{e}^i\right)^{\gamma-\alpha_E} = \frac{A\alpha_E\kappa_i\overline{k}^{\alpha_K}}{g\gamma}$. Rearranging, we get

$$0 < \left.\left(A\alpha_E\kappa_i\left(\frac{\overline{k}}{E}\right)^{\alpha_K} - g\gamma E^{\gamma-1}\right)\right|_{T^b} \tag{C.8}$$

The RHS of equation $(C.8)$ is nothing else than the term in the *square bracket* of equation (C.4) implying $\frac{dU^i}{dT} < 0$ on the interval $(\widehat{T}^i, \infty)$. Hence $U^i(t,T)$ is a concave function with a unique maximum at $\widehat{T}^i > 0$. Consider $\frac{\partial \widehat{e}^i}{\partial \kappa_i}$ to see that the federal taxes can be ranked such that $\widehat{T}^n < ... < \widehat{T}^1$. $\square$

Appendix D. Proof of Proposition 7

All else equal as in Appendix C except for the assumption that each state government takes into account how its policy influences the federal policy $\left(\frac{\partial \Pi_i}{\partial \tau_i} \neq 0\right)$ and $\gamma = 1$. We get

$$t_i(T) = g\gamma E^{\gamma-1} - \kappa_i T, \tag{D.1}$$

$$E_i(t,T) = \left(\frac{A\alpha_E \overline{k_i}^{\alpha_K}}{g\gamma E^{\gamma-1} + (1-\kappa_i)T}\right)^{\frac{1}{\alpha_K}},$$

$$E(t,T) = \sum_i \left(\frac{A\alpha_E \overline{k_i}^{\alpha_K}}{g\gamma E^{\gamma-1} + (1-\kappa_i)T}\right)^{\frac{1}{\alpha_K}}, \tag{D.2}$$

and

$$U^i(t,T) = \frac{g - \kappa_i T + \alpha_K T}{\alpha_E} E_i + (\kappa_i T - g_i) E. \tag{D.3}$$

The derivative of U^i with regard to T is

$$\frac{dU^i}{dT} = \frac{\alpha_K - \kappa_i}{\alpha_E} E_i - \frac{g + (\alpha_K - \kappa_i)T}{\alpha_E \alpha_K} \left(\frac{1}{g_i + (1-\kappa_i)T}\right)^{\frac{1+\alpha_K}{\alpha_K}} \left(A\alpha_E k_i^{\alpha_K}\right)^{\frac{1}{\alpha_K}} + \kappa_i E + \kappa_i T \frac{\partial E}{\partial T} - \frac{\partial (gE^{\gamma})}{\partial T} \tag{D.4}$$

Evaluate $\frac{dU^i}{dT}$ at $T = 0$ to get

$$\left.\frac{dU^i}{dT}\right|_{T=0} = \left(\alpha_K - \kappa_i - \frac{1}{\alpha_K}\right) \frac{1}{\alpha_E} \left(\frac{A\alpha_E k_i^{\alpha_K}}{g}\right)^{\frac{1}{\alpha_K}} + \left(\frac{1}{\alpha_K} + \kappa_i\right) \left(\frac{A\alpha_E k^{\alpha_K}}{g}\right)^{\frac{1}{\alpha_K}}. \tag{D.5}$$

Rearranging and substituting $\widetilde{e}_i = \left(\frac{A\alpha_E k_i^{\alpha_K}}{g}\right)^{\frac{1}{\alpha_K}}$ and $\widetilde{e} = \left(\frac{A\alpha_E k^{\alpha_K}}{g}\right)^{\frac{1}{\alpha_K}}$ into equation $(D.5)$ we obtain

$$0 < \left(\alpha_K - \kappa_i - \frac{1}{\alpha_K}\right) \kappa_i + \alpha_E \left(\frac{1}{\alpha_K} + \kappa_i\right). \tag{D.6}$$

The restriction $\kappa_i < \frac{\alpha_K \kappa_i + \alpha_E}{\alpha_K \kappa_i + 1}$ ensures that $(D.6)$ holds and hence $U^i > \widetilde{U}^i$.

$\square$