Assessment of proxy-hedging in jet-fuel markets

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Abstract

The aim of this research is to explore the risk associated with hedging in jet fuel markets. It focuses on finding the most effective proxy hedge instrument for the Singapore spot market. Due to its particularities, this market does not exhibit the same features as traditional financial markets do. In appearance it seems very related to the oil market, but in reality it exhibits insufficient liquidity and shows unusual volatility clustering effects. This behavior has a direct impact on the hedging strategies of refineries, airline companies and jet fuel traders. The paper explores the econometric features of the jet fuel price and underlines the need of fat tail distributions and volatility clustering models. Also it examines the density forecasting capacity of various proxy hedge instruments including kerosene, crude and gasoil futures. The results show that Singapore Gasoil Futures contract is the best candidate for hedging the Singapore Jet Fuel spot price.

Keywords: Oil distillates, Gasoil, Jet Fuel, hedging strategies, market liquidity, market efficiency

1. Introduction

An extensive literature covers the economy of oil markets, but less attention is given to the oil distillates and particularly to the jet fuel market. The lack of efficiency in oil and middle distillates markets was pointed previously by the academic literature (Balbás et al. (2008), Kanamura et al. (2010), Roncoroni et al. (2015)). Oil distillates markets are by their nature dependent on the oil market behavior, but also are exposed to specific risks linked to the changes in the supply/demand equilibrium for those products. Therefore, one avenue we intend to explore in this research is the modeling based on non-Gaussian distribution, volatility clustering and regime switching.

The main motivation however behind this study is to address the challenges faced by a company trading illiquid refined products such as jet fuel and providing it with optimal solutions with regards to their proxy hedging. Nascimento and Powell (2008) modeled the jet fuel price using two-factor model to allow mean-reversion in the short-term and proposed oil future contracts for tackling the hedging problem. Adams and Gerner (2012) investigated the effect of the maturity on the cross-hedging performance of jet-fuel within an Error Correction model. They evaluated the performance of several oil forwards contracts including WTI, Brent, Gasoil and heating oil to manage jet-fuel spot price exposure. Their results highlight that the standard approach in the literature to use crude oil as a cross hedge for jet fuel is not optimal for time horizons of three months or less. By contrast, for short hedging horizons their results indicate that gasoil forwards contracts represent the highest cross hedging efficiency for jet-fuel spot price exposure, while for maturities of more than three months, the predominance of gasoil diminishes in comparison to WTI and Brent.

Clark et al. (2003) have attempted to test for the most effective cross hedging instrument for the Singapore jet fuel spot market, using regression techniques. Their research concludes that for the period February 1997 to August 2001, Heating Oil futures contract gives best in sample results. Nevertheless, after correcting for serial correlation , their out of sample results proved to be weak for all regression models and ambiguous with respect to the heating oil contract.

This paper aims to enrich the scarce literature on the economics of oil distillates and attempts to estimate a good model capturing the dynamics of jet fuel futures. In contrast to level forecasting regression and co-integration models used in previously mentioned papers, our research provides a different approach for testing proxy-hedging based on density forecasting. The paper is organized as follows :

- **Section 2** explores the econometric features of oil middle distillates refined products (including gasoil and jet fuel)
- **Section 3** explains the challenges of jet fuel proxy-hedging as well as the associated basis risk
- **Section 4** assesses the density forecasting methodologies (including probability forecasting Gneiting Test Gneiting and Ranjan (2011))
- **Section 5** presents the results of the ability of more liquid traded products such as Brent Crude and Gasoil returns to forecast density the jet fuel market
- **Section 6** concludes

2. Econometric modeling of the Singapore jet fuel and related oil distillates

The first part of this research is dedicated to the econometric study of the Singapore jet fuel and related oil distillates prices. Our aim is not to find the "true" model that would explain the behavior of these commodities, but to propose a benchmark from different models commonly used to describe financial assets. Based on the historical time series, few models are estimated with the intention to capture volatility clustering. Clustering in volatility is another ubiquitous feature observed in returns. Few models from the GARCH universe allow to capture this phenomena emphasizing the various particularities of the return series.

We explore the following models :

— Models without volatility clustering, but with non Gaussian innovations (NIG, t-Student, Asymmetric Student)

— Models with volatility clustering and Gaussian innovations (GARCH, eGARCH, iGARCH, GJR-GARCH, APARCH)

— Models with volatility clustering and non-Gaussian innovations

— Markov Regime Switching GARCH models

Looking forward lets consider that S_t the asset price at time t has the following dynamics under the empirical measure $\mathbf{P}$:

$$Y_t = ln\frac{S_t}{S_{t-1}} = r + \psi\sqrt{h_t} + \epsilon_t \tag{1}$$

$$Y_t = ln\frac{S_t}{S_{t-1}} = r + \psi\sqrt{h_t} + \sqrt{h_t}\cdot z_t \tag{2}$$

where ϵ_t has zero mean and conditional variance h_t under the measure $\mathbf{P}$;z_t is a i.i.d. distributed variable ; r is the one-period risk-free rate of return and ψ the constant unit risk premium.

2.1. Dataset presentation

As emphasized earlier, the final goal of this article is to assess the risk of a refinery or airline company that hedges its exposure to illiquid petroleum products such as jet fuel. There are two primary futures contracts which are commonly used for jet fuel hedging : brent crude and gasoil. These contracts serve as the primary benchmarks across the globe. In addition, there are many other contracts (futures, crack futures, swaps and options) available for jet fuel hedging, most of which are tied to one of the major, global trading hubs of Singapore, US Gulf Coast (Houston/New Orleans) and NW Europe/ARA (Amsterdam, Rotterdam and Antwerp).

FIGURE 1: Evolution of the front month futures of Singapore Jet Kerosene , ICE Brent Crude, ICE Low Sulphur Gasoil and Singapore 50ppm Gasoil (USD/bbl)

For this purpose, we consider ICE Brent Crude, ICE Low Sulphur Gasoil and Singapore 50ppm Gasoil Futures for our proxy analysis. We also consider the Singapore Jet Kerosene (Platts) vs. Gasoil (Platts) Futures differential (called Regrade), often used in jet fuel hedging. As for the jet fuel, there are three reference futures contracts for each geographical hub : Platts CIF NWE, USGC Jet 54 and Singapore Jet FOB. For our analysis, we consider the jet fuel contract traded in Singapore. Figure 1 [1] presents the evolution of the above mentioned front month futures contract quoted in

1. The ICE Low Sulphur Gasoil contract, quoted in USD per metric tones on the exchange, has been converted here to USD/bbl using a scale conversion factor of 7.45 used in the industry.

USD/barrel.

The Table 2.1 synthesizes the summary statistics over the considered dataset. ICE brent and ICE Low Sulphur Gasoil exhibit a higher volatility compared to the other three series. We notice that compared to the highly liquid Brent and LS Gasoil futures, Singapore Jet Fuel/Kerosene, Singapore Gasoil and Regrade exhibit a considerably higher kurtosis values which implies the need of heavy tailed distributions for modeling purposes.

Underlying	**Mean**	**Volatility**	**Skewness**	**Kurtosis**
Regrade	-0.003	0.287	0.803	34.157
Singapore Gasoil	-0.001	0.289	-0.374	15.039
Singapore Kerosene	-0.001	0.265	-0.338	22.417
ICE Brent	-0.001	0.390	0.248	5.134
ICE LS gasoil	-0.001	0.328	0.597	6.862

Table 1: Summary Statistics. ICE brent and ICE Low Sulphur Gasoil exhibit a higher volatility compared to the other three series. Regrade and Singapore Kersone have a more pronounced Kurtosis.

Figures 8, 11, 14, 17 and 20 show the historical prices for Brent, LS Gasoil, Jet Kerosene and Regrade futures for the most liquid maturities of the curve.

2.2. Generalized Hyperbolic models

A recent modeling technique introduced here permits both skewness and kurtosis in the assets returns. Indeed, these features are not accounted for in the previous modelings. Following the works of Eberlein and Prause (2002) and Barndorff-Nielsen (1977) done on financial assets, we calibrate the class of Generalized Hyperbolic distributions to our data sets. This very flexible class of distributions (Annexe 1) is able to capture heavy tails and asymmetry. It is characterized by five parameters with a parameter which permits very specific shapes. The four other parameters are linked in an easy way with the first four moments of the distribution.

2.2.1. Distributions Fit Results

In order to add leptokurtic distribution shapes of our datasets and overpass the limitations of using the classic Gaussian modeling framework, we consider the following set of candidate distributions : t-Student, Assymetric Student and Normal Inverse Gaussian (NIG), which retained our attention for their capacity to take in account heavy tails. The results of the statistical estimation are exhibited in the following tables. The 95% confidence intervals are compute through bootstrap approaches. The fittings are compared based upon the Bayesian Information Criterion (BIC) [2]. The results synthesized in Tables 2.2.1, 2.2.1, 4, 2.2.1 and 2.2.1 Student and NIG distributions exhibit the best fits for the jet fuel returns as well as for the proxy-hedging candidates.

Gaussian		**Student**		**SSTD**		**NIG**	
Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
μ	0.000	μ	-0.001	μ	0.000	μ	-0.000
	[0.000 0.000]		[0.000 0.000]		[0.000 0.000]		[0.000 0.000]
σ	0.025	σ	0.026	σ	0.026	α	0.025
	[0.023, 0.026]		[0.024, 0.027]		[0.024, 0.027]		[0.023, 0.026]
		ν	3.529	β	1.071	β	0.115
			[3.4, 3.6]		[1.061, 1.081]		[0.11, 0.12]
				ν	3.550	δ	0.676
					[3.440,3.660]		[0.442, 0.852]
BIC	-1724.58		-1760.64		-1724.82		**-1764.04**

TABLE 2: Distribution Fitting for ICE Brent Front Month Futures returns. NIG and Student distribution exhibit the best fits.

2.3. *Volatility models*

Typical Gaussian flat volatility failed to provide with conspicuous valuations for contingencies and also underestimated the risk measures. The dynamic volatility models add value also for testing hedging strategies as, the traditional flat volatiles model tend to underestimate the clustering effect. For this purpose we consider the GARCH-type models. The GARCH process introduced by Bollerslev (1987) and its different variations have gained increasing prominence for modelling financial asset over the last decade. The GARCH diffusion presents three particular features compared to other modellings. First it assumes that the present conditional variances is linearly linked to the past conditional variances and to past market squared return. Second for an accurate calibration GARCH is greedy in term of data. Third the models transfers through volatility pastern the risk premium of the underlings price. The classic GARCH

2. In our formalism the higher the absolute value of the BIC, the better the fit is.

Gaussian		Student		SSTD		NIG	
Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
μ	-0.000	μ	-0.001	μ	-0.001	μ	0.000
	[-0.230 ,0.220]		[-0.203,0.323]		[-0.101,0.14]		[-0.120,0.122]
σ	0.021	σ	0.028	σ	0.028	α	0.021
	[0.019,0.023]		[0.024,0.033]		[0.025, 0.032]		[0.019,0.022]
		ν	2.468	β	1.001	β	0.064
			[2.325, 2.514]		[0.952, 1.053]		[0.042, 0.086]
				ν	2.469	δ	0.349
					[2.221,2.66]		[0.247, 0.424]
BIC	-1856.41		-1922.24		-1880.75		**-1923.88**

Table 3: Distribution Fitting for ICE LS Gasoil Front Month Futures daily returns. NIG and Student distributions exhibit the best fits.

Gaussian		Student		SSTD		NIG	
Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
μ	-0.000	μ	0.000	μ	-0.000	μ	0.000
	[-0.210 ,0.215]		[-0.213,0.223]		[-0.11,0.12]		[-0.110,0.132]
σ	0.017	σ	0.066	σ	0.066	α	0.012
	[0.014,0.023]		[0.028,0.088]		[0.038,0.091]		[0.002,0.021]
		ν	2.010	β	0.938	β	-0.112
			[1.968,2.13]		[0.842,1.201]		[-0.154,-0.049]
				ν	2.010	δ	0.100
					[1.840,1.260]		[0.042, 0.152]
BIC	-2011.99		**-2260.43**		-2192.86		-2245.14

Table 4: Distribution Fitting for Singapore Jet Fuel/Kerosene Front Month Futures daily return. Student and NIG distributions exhibit the best fits.

framework bring obviously significant improvements in term of econometric description compared to the classic Gaussian model. And yet Bollerslev's GARCH remains still under an assumption of normally distributed innovations.

Further under the framework described by Bollerslev (1987), ϵ_t follows a GARCH(1,1) process is

$$\epsilon_t|\phi_{t-1} \propto N(0, h_t) \ or \ z_t \propto N(0, 1) \tag{3}$$

$$h_t = \alpha_0 + \alpha_1 \cdot \epsilon_{t-1}^2 + \beta_1 \cdot h_{t-1} \tag{4}$$

Gaussian		Student		SSTD		NIG	
Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
μ	0.000	μ	0.000	μ	-0.000	μ	0.000
	[-0.20 ,0.205]		[-0.23,0.22]		[-0.112,0.122]		[-0.110, 0.101]
σ	0.018	σ	0.082	σ	0.082	α	0.015
	[0.015,0.021]		[0.059,0.118]		[0.041,0.128]		[0.09,0.021]
		ν	2.010	β	0.958	β	-0.079
			[1.64,2.48]		[0.847,1.131]		[-0.104,-0.045]
				ν	2.010	δ	0.100
					[1.740,2.460]		[0.042, 0.152]
BIC	-1949.60		**-2151.21**		-2082.55		-2141.86

TABLE 5: Distribution Fitting for Singapore Gasoil Front Month Futures returns. Student and NIG distributions exhibit the best fits.

Gaussian		**Student**		**SSTD**		**NIG**	
Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
μ	-0.000	μ	-0.003	μ	-0.005	μ	-0.001
	[-0.22 ,0.205]		[-0.28,0.2]		[-0.112, 0.122]		[-0.10, 0.111]
σ	0.287	σ	0.911	σ	0.911	α	0.181
	[0.265,0.312]		[0.88,1.08]		[0.85,1.02]		[0.184,0.221]
		ν	2.010	β	0.983	β	0.038
			[1.842, 2.268]		[0.652,1.201]		[0.018,0.054]
				ν	2.010	δ	0.100
					[1.540,2.560]		[0.042, 0.152]
BIC	128.943		**-177.201**		-117.776		-156.712

TABLE 6: Distribution Fitting for Regrade Front Month Futures returns. Student and NIG distributions exhibit the best fits.

where ϕ_t is the corresponding σ-algebra generated by the previous an present information ; The unconditional variance is $h_0 = \frac{\alpha_0}{(1-\alpha_1-\beta_1)}$. GARCH model assumes that the conditional variance is a linear function of past squared disturbances and the past conditional variance, genuinely making h_t ϕ_t-predictable.

Generalizing the above given definition a GARCH(p,q) follows

$$h_t = \alpha_0 + \alpha_1\epsilon_{t-1}^2 + \cdots + \alpha_q\epsilon_{t-q}^2 + \beta_1 h_{t-1} + \cdots + \beta_p h_{t-p}^2 = \alpha_0 + \sum_{i=1}^{q}\alpha_i\epsilon_{t-i}^2 + \sum_{i=1}^{p}\beta_i h_{t-i}$$

where $p \geq 0$, $q \geq 0$, $\alpha_0 > 0$, $\alpha_i > 0$, i=1,...,q; $\beta_i \geq 0$, i=1,...,p. and $\sum_{i=1}^{q}\alpha_i + \sum_{i=1}^{p}\beta_i < 1$ [3].

In order to mitigate the existence of significant kurtosis and skewness effects assets returns returns an extension of the GARCH model could be the introduction of non-Gaussian (Generalized Hyperbolic) innovations, with the parametrization introduced in the previous section :

$$z_t \propto GH(\lambda; \alpha; \beta; \mu; \delta) \ or \tag{5}$$

$$\epsilon_t|\phi_{t-1} \propto GH(\lambda; \frac{\alpha}{\sqrt{h_t}}; \frac{\beta}{\sqrt{h_t}}; \mu\sqrt{h_t}; \delta\sqrt{h_t}) \tag{6}$$

$$h_t = \alpha_0 + \alpha_1 \cdot \epsilon_{t-1} + \beta_1 \cdot h_{t-1} \tag{7}$$

GARCH diffusion presents in term of pricing three particular features compared to other modellings. First the GARCH derivatives prices depends of risk premium embedded in the underlying asset. Second the GARCH pricing model is non-Markovian and is an interesting alternative for markets with serial dependency. Third the GARCH models might explain some valuation biases of out-of the money options, associated with classic models.

Few popular variations of the GARCH model include :

— **The integrated GARCH (IGARCH) model** The integrated GARCH model (Engle and Bollerslev (1986)) assumes that the persistence is one. Omitted structural breaks should be assessed before using an iGARCH model.

$$\epsilon_t|\phi_{t-1} \propto N(0, h_t) \ or \ z_t \propto N(0,1) \tag{8}$$

$$h_t = \alpha_0 + (1-\beta_1) \cdot \epsilon_{t-1}^2 + \beta_1 \cdot h_{t-1} \tag{9}$$

— **The Glosten-Jagannathan-Runkle(GJR-GARCH) model** introduced by Glosten et al. (1993) adds assymetry in the volatility process :

3. For insuring the covariance stationarity of the GARCH(p,q) it is imposed that the persistence is inferior to the unity

$$h_t = \alpha_0 + (\alpha_1 + c \cdot I_{t-1}) \cdot \epsilon_{t-1}^2 + \beta_1 \cdot h_{t-1} \tag{10}$$

where

$$I_{t-1} = \begin{cases} 0 & \epsilon_{t-1} \geq 0 \\ 1 & \epsilon_{t-1} < 0 \end{cases}$$

— **The exponential GARCH (EGARCH) model** introduced by Nelson (1991) aims to capture asymmetric reaction of volatility to the positive and negative information about the market. Volatility of the EGARCH model, which is measured by the conditional variance is an explicit multiplicative function of lagged innovations.

$$\log h_t = \alpha_t + \sum_{i=1}^{\infty} \beta_i g(Z_{t-k}) \tag{11}$$

where the function g is defined as $g(Z_t) = \theta Z_t + \gamma(|Z_t| - E|Z_t|)$, $g(Z_t)$ having a zero mean $E[g(Z_t)] = 0$. No restriction are imposed in this version of the GARCH model. EGARCH can also assess whether the shocks in variance are persistent or not.

— **The Asymmetric Power GARCH model (APARCH)** introduced by Ding et al. (1993) accounts for leverage effect and also the fact that the sample autocorrelation of absolute returns is higher than that of squared returns(Reider (2009)).

$$h_t^{0.5 \cdot \zeta} = \alpha_0 + \sum_{i=1}^{q} \alpha_i (|\epsilon_{t-i}^{\zeta}| - \gamma_i \cdot \epsilon_{t-i})^{\zeta} + \sum_{i=1}^{p} \beta_i h_{t-i}^{0.5 \cdot \zeta} \tag{12}$$

It can be notice that equation 12 with ζ=2 and $\gamma_i = 0$ matches the classic GARCH model with Gaussian innovations.

We estimated through max-likelihood method the volatility models presented above. Table 7 exhibits the results of fitting of GARCH-type model with normal, Student and NIG innovations for Singapore Jet Fuel daily returns. The models with NIG innovation show a superior fitting performance in terms of BIC /AIC. APARCH model fits better than the rest of the GARCH family for all three innovation type. The APARCH with NIG innovations and with a power factor (δ) of 0.94 exhibits the best features, underlining the leverage effects in volatility.

Model	Normal				STD				NIG			
	Param.	Estimate	Std. Error	p-value	Param.	Estimate	Std. Error	p-value	Param.	Estimate	Std. Error	p-value
GARCH												
	ω	0.000	0.000	0.000	ω	0.000	0.000	0.007	ω	0.000	0.000	0.001
	α_1	0.002	0.000	0.000	α_1	0.347	0.061	0.000	α_1	0.307	0.055	0.000
	β_1	0.994	0.000	0.000	β_1	0.652	0.064	0.000	β_1	0.692	0.043	0.000
					ν	2.314	0.074	0.000	α	-0.228	0.047	0.000
									β	0.055	0.009	0.000
BIC/AIC	-5.351	-5.333			-6.714	-6.690			**-6.770**	**-6.739**		
eGARCH												
	ω	-0.040	0.004	0.000	ω	-0.904	0.265	0.001	ω	-0.642	0.201	0.001
	α_1	-0.061	0.018	0.001	α_1	-0.004	0.040	0.911	α_1	-0.005	0.066	- 0.937
	β_1	0.994	0.000	0.000	β_1	0.895	0.027	0.000	β_1	0.907	0.026	0.000
	γ_1	0.033	0.013	0.011	γ_1	0.558	0.057	0.000	γ_1	0.886	0.259	0.001
					ν	2.100	0.037	0.000	α	-0.264	0.049	0.000
									β	0.010	0.006	0.079
BIC/AIC	-5.424	-5.399			-6.677	-6.646			**-6.776**	**-6.740**		
iGARCH												
	ω	0.000	0.000	0.900	ω	0.000	0.000	0.000	ω	0.000	0.000	0.000
	α_1	0.000	0.001	0.995	α_1	0.348	0.042	0.000	α_1	0.308	0.039	0.000
	β_1	1.000			β_1				β_1	0.692		
					ν	2.313	0.040	0.000	α	-0.229	0.046	0.000
									β	0.055	0.008	0.000
	-5.355	-5.343			-6.717	-6.698			**-6.773**	**-6.748**		
gjrGARCH												
	ω	0.000	0.000	0.010	ω	0.000	0.000	0.071	ω	0.000	0.000	0.001
	α_1	0.000	0.005	1.000	α_1	0.314	0.089	0.000	α_1	0.274	0.066	0.000
	β_1	0.980	0.002	0.000	β_1	0.649	0.098	0.000	β_1	0.690	0.044	0.000
	γ_1	0.032	0.010	0.001	γ_1	0.073	0.127	0.566	γ_1	0.081	0.109	0.459
					ν	2.318	0.097	0.000	α	-0.231	0.047	0.000
									β	0.055	0.009	0.000
	-5.406	-5.382			-6.712	-6.681			**-6.768**	**-6.731**		
APARCH												
	ω	0.001	0.000	0.000	ω	0.000	0.000	0.520	ω	0.003	0.005	0.573
	α_1	0.007	0.002	0.001	α_1	1.000	0.335	0.003	α_1	1.000	0.229	0.000
	β_1	0.991	0.000	0.000	β_1	0.636	0.040	0.000	β_1	0.670	0.054	0.000
	γ_1	1.000	0.001	0.000	γ_1	0.093	0.080	0.245	γ_1	0.081	0.109	0.455
	δ	0.432	0.038	0.000	δ	1.646	0.319	0.000	δ	0.939	0.365	0.010
					ν	2.128	0.053	0.000	α	-0.302	0.049	0.000
									β	0.014	0.006	0.016
	-5.538	-5.507			-6.748	-6.711			**-6.833**	**-6.790**		

TABLE 7: Fitting of GARCH-type model with normal, Student and NIG innovations for Singapore Jet Fuel daily returns

2.3.1. *Markov Regime Switching GARCH models*

Despite adding value for modeling assets with leptokurtoic behavior single regime GARCH models, fail to capture time of a transition between between a low risk and high risk regime. An alternative was introduced by Haas et al. (2004) with the switching regime GARCH model detailed din the below formula. Middle distillates markets are particularly concerned by this feature due to the variation in liquidity. Thus one volatility regime can correspond to thin liquidity conditions while another to appropriate levels of liquidity. The GARCH switching regime is specifies as following :

$$h_t = \begin{cases} \alpha_0^1 + \alpha_1^1 \cdot \epsilon_{t-1}^2 + \beta_1^1 \cdot h_{t-1}; \\ \alpha_0^2 + \alpha_1^2 \cdot \epsilon_{t-1}^2 + \beta_1^2 \cdot h_{t-1}; \end{cases} \tag{13}$$

The results of fitting the switching GARCH model for the underlyings studied in this paper are exhibited in Table2.3.1. The occurrence of two distinct states with statistically significant probability of transition is confirmed for Singapore Gasoil and Regrade. The particularity of this two underlying is the fact that they trade on thinner liquidity than the other three markets considered in this study. This finding confirms our initial assumption and is a valuable learning when testing the risk related to proxy hedging.

	State1			**State2**				
	α_{01}	α_{11}	β_1	α_{02}	α_{12}	β_2	P_1	P_2
Regrade	0.0008	0.1677	0.3855	0.1053	0.0266	0.7489	0.8670	0.8546
Sing GO	0.0001	0.0001	0.0001	0.0001	0.0001	0.9484	0.9532	0.2853
Kerosene	0.0001	0.0423	0.0001	0.0008	0.0001	0.7934	0.9442	0.9145
Brent	0.0001	0.2862	0.5219	0.0005	0.0016	0.8411	0.9673	1.0000
LSGO	0.0001	0.0004	0.0001	0.0001	0.0863	0.7308	0.9946	0.0000

TABLE 8: Switching Regime GARCH models fitting for ICE Brent, ICE Low Sulphur Gasoil, Singapore Gasoil, Jet Fuel/Kerosene and Regrade

3. Proxy hedging

In an incomplete financial market, it is hardly ever possible to find the hedging instrument that perfectly mirrors a given price risk. Most of refiners and airline companies attempt to hedge around 80% of their jet fuel exposure (Adams and Gerner (2012)). In large part, they do this by purchasing futures contracts on crude oil, the feedstock for producing jet fuel, or other oil derivative products such as heating oil, used in USA, and gasoil in Europe. The regional prices of these commodities with jet fuel are correlated in the long run, however in the short term, price co-movements are asynchronous. This erratic relationship defines the basis risk.(Kamara and Siegel (1987) ,Ankirchner and Imkeller (2011)), which is the financial risk occurred when the chosen 'proxy-hedge' does not entirely offset the price risk of the main underlying asset.

This is clearly seen from the 1 Month rolling correlation plot in Figure 2, which shows that despite the 'obviously' highly correlated dynamics of the spot prices exhibited earlier in Figure 1, there exists basis risk when hedging in oil markets and this is mainly explained by product, location and time factors.

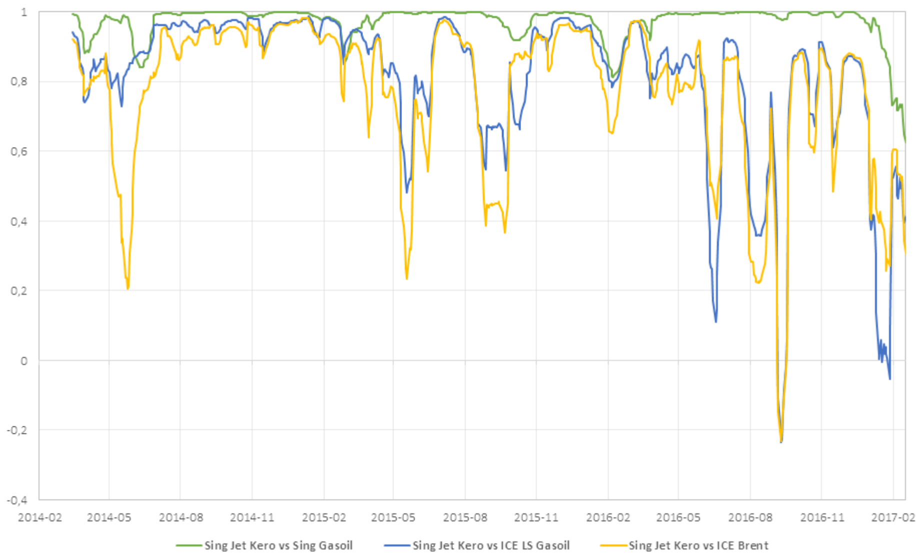

FIGURE 2: 1M Rolling Correlation of Front Month Futures

Another representation of the basis risk can be seen in Figure 20 which shows the price the differential between Singapore Gasoil and Jet Kerosene, or the Regrade. All this is to show that even when a proxy instrument appears to be a highly correlated instrument, the basis risk associated with it undermines the effectiveness of the proxy-hedge as it exacerbates the cash flow volatility that the hedge is designed to reduce.

A major challenge in "proxy hedging" (Viken and Thorsrud (2014)) consists in finding an the proxy instrument which will minimize basis risk and hedge volatility. As such, an Asian company willing to cover it's long jet fuel exposure has the following

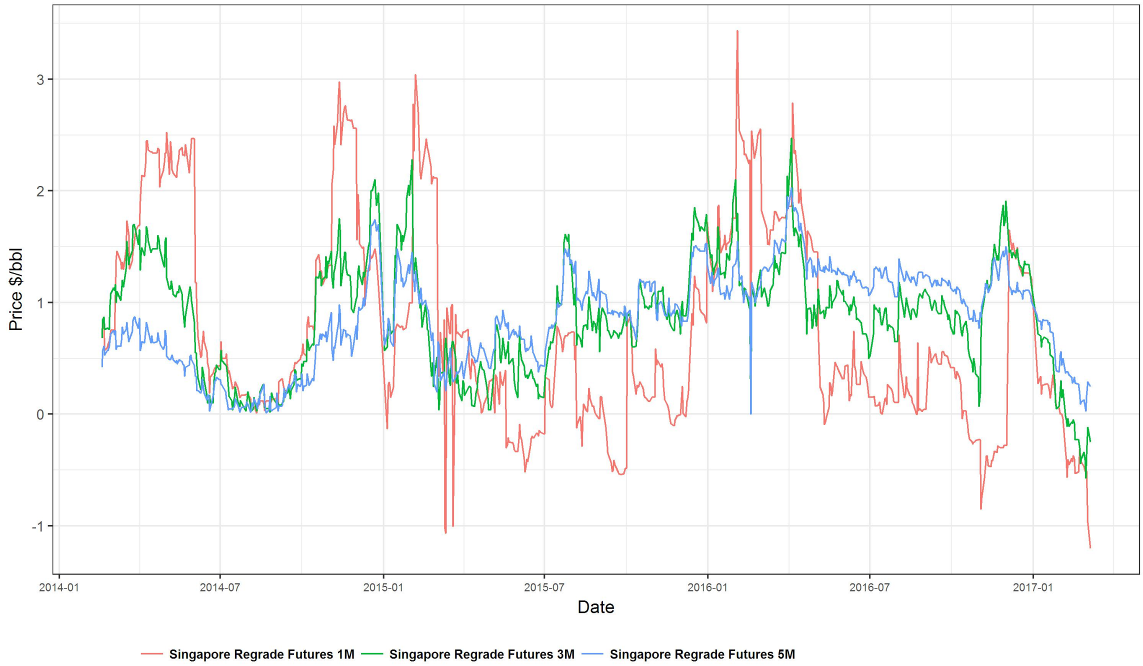

FIGURE 3: Singapore Regrade Futures

options :

- **Jet Fuel direct hedge** : sell Singapore Jet Kerosene Futures or first line swap
- **Crude 'proxy hedge'** : sell ICE Brent Crude Futures or first line swap
- **Gasoil 'proxy hedge'** : sell ICE LS Gasoil or Singapore 0.5% Gasoil Futures
- **Basis risk 'slice & dice proxy hedge'** : hedge jet fuel pricing components opportunistically in case of an existing exposure to crude or gasoil (use Brent/Gasoil Crack, Regrade or Jet Differential)

Now the main question which stands out, is why hedge using crude oil or gasoil contracts when jet fuel future contracts are also available ? The answer is contract liquidity. Figure 3 exhibits volumes of the considered dataset on a logarithmic scale. We notice that ICE Brent Crude oil and LS Gasoil futures are significantly more liquid than the Singapore Kerosene and Gasoil 0.5% futures. If jet fuel contracts were available at the same cost as crude oil contracts, then clearly this would be a better alternative.

As such, if an Asian airline company wants to cover its jet fuel price risk, since the volumes exchanged on this market are thin, it might use one of the 'proxy-hedge' options described earlier. However, choosing the right one means making a trade-off between liquidity and basis.

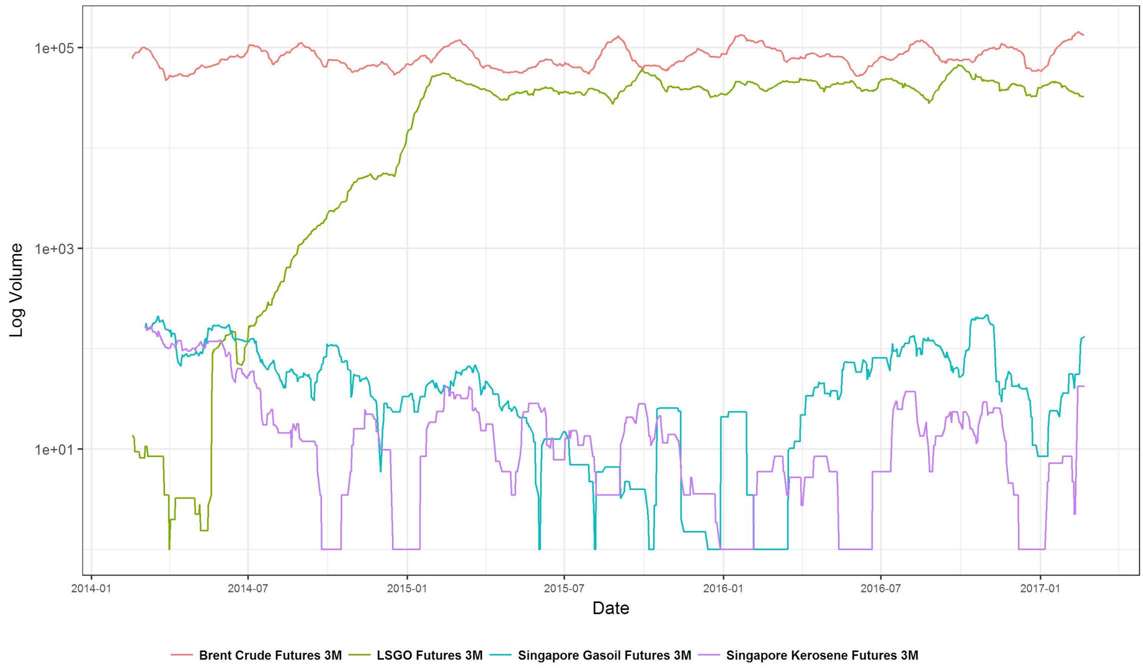

FIGURE 4: ICE Brent, ICE LS Gasoil, Singapore Gasoil and Jet Fuel Futures Liquidity

The current literature focuses mainly on the risks related to level forecasting when using a proxy-hedge, but ignores completely the density forecasting. The main issue with proxy hedging is the fact that markets have different depth. On one hand a shock in the Brent market might not be fully reflected in the Jet fuel market. On the other hand a small variation in the Brent Market might generate a shock in the Jet fuel market due to difference in liquidity. The basis risk of proxy hedge using both plain or derivatives based strategies is generated also by the differences in the distribution features thereby underlying the need of testing the density forecasting ability.

For testing the proxy-hedging with Brent, gasoil or regrade, a trader exposed to jet fuels prices risk should assess the density forecasting capacity of an econometric risk model. Thus a model estimated on Brent or Gasoil returns should be tested in terms of density forecasting on the jet fuel prices.

4. Forecasting densities

This section describes the technique for reaching the main goal of this paper, the testing in terms of density forecasting of proxy-hedging strategies. In a recent paper

Gneiting and Ranjan (2011) proposed a test that develops the weighting approach of Amisano and Giacomini (2007) but avoids counter intuitive inferences. We use this test for assessing the density forecasting in proxy hedging.

Gneiting's test aims to built a proper score with the respect of the above definition based on appropriately weighted versions of the continuous ranked probability score(CPRS). For any density function f(y) with a cumulative distribution function $F(z) = \int_{-\infty}^{z} f(y)dy$the continuous ranked probability score is then defined as

$$CPRS(F, y) = \int_{-\infty}^{\infty} PS(F(r), 1(y \leq r))dr \tag{14}$$

where

$$PS(F(r), 1(y \leq r)) = (1(y \leq r) - F(r))^2 \tag{15}$$

is the Brier probability score for the probability forecast $F_t(r) = \int_{-\infty}^{r} f(y)dy$ of the event $y \leq r$

The weighted probability score described by Matheson and Winkler (1976) and Gneiting and Raftery (2007) is written as :

$$S^w(f, y) = -\int_{-\infty}^{\infty} PS(F(r), 1(y \leq r)w_r(r)dr \tag{16}$$

where the weighting function $w_r(r)$ taxes the forms presented in equation **??** In a discrete form the above score can be aproximtd by assuming an equidstan discretizations of a target region with the boundaries y_l, y_u

$$S_f^w(f, y) = \frac{y_u - y_l}{I - 1} \sum_{i=1}^{I} w(y_i)PS(F(y_i), I(y \leq y_i)) \tag{17}$$

$y_i = y_l + i\frac{y_u - y_l}{I}$ The test based on the following statistic which is leveraged from the Amisano-Giacomini test :

$$Z_n = \frac{\mathbf{E}(S_f^w(f, y) - S_f^w(g, y))}{\widehat{\omega_n}} \tag{18}$$

where

$$\mathbf{E}_t(S_f^w(f, y) = \frac{1}{n - k + 1} \sum_{t=m}^{m+n-k} S(f_{t+k}, y_{t+k}) \tag{19}$$

$$\mathbf{E}_t(S_f^w(g, y) = \frac{1}{n - k + 1} \sum_{t=m}^{m+n-k} S(g_{t+k}, y_{t+k}) \tag{20}$$

and $\widehat{\omega_n}$ is an estimate of $var(\sqrt{n}(\mathbf{E}_t(S_f^w(f, y) - \mathbf{E}_t(S_g^w(g, y))$

5. Backtesting results of proxy-hedging

Based on the specifications of the Gneitting test presented above we built a testing process for the proxy hedging strategies. The full dataset contains the daily prices of Jet-fuel, ICE Brent, ICE gasoil or Singapore Gasoil between 01/01/2014 and 01/03/2017. The testing process has the following steps :

1. A model (M1) is estimated on the daily returns of the proxy (ICE Brent, ICE gasoil or Singapore Gasoil). The data set contains a (*out of sample*) the first 250 consecutive days of the considered full sample.
2. A model (M2) is estimated on the daily returns of the jet fuel prices. The data set (*in sample*) contains a window of 250 consecutive days, which starts immediately after the end of the out of sample dataset.
3. The Gneitting test score is computed for comparing the model M1 estimated *out of sample* on the proxy with model M2 estimated on the actual *in sample* jet fuel returns
4. The *out of sample* window is rolled over with one day and same is for the *in sample* window. Steps 1-3 are repeated until the end of the full sample
5. A time series of Gneitting test scores is built.

The previous sections underlined that NIG distribution exhibits good fitting features for all the underlying studied in this article. Therefore we will consider the NIG model for both *out of sample* and *in sample*. Therefore the test score will assess the power of the density fitted on the proxy to forecast the jet fuel distribution feature.

Figure 5 shows the evolution of the testing Score for NIG model, where as the proxy hedge is realized with Singapore Gasoil. Until July 2016 the score rejects at 99% confidence level the null hypothesis that the model fitted on the proxy is similar to the model fitted on the jet fuel and in fact the proxy provides with better results. After July 2016 the score enter in the confidence region thereby not rejecting the null hypothesis. Towards 2017 the NIG model fitted on proxy losses gradually from its forecasting capacity but remains close to the confidence region

Figure 6 shows the evolution of the testing Score for NIG model, where as the proxy hedge is realized with ICE Low Sulphur Gasoil. Until July 2016 the score does not rejects at 99% confidence level the null hypothesis that the model fitted on the proxy

FIGURE 5: Evolution of the Gneitting Test Score for NIG model with Singapore Gasoil

is similar to the model fitted on the jet fuel. After July 2016 the NIG model fitted on Low Sulphur Gasoil proxy has lost its forecasting capacity and became inappropriate.

Figure 7 shows the evolution of the testing Score for NIG model, where as the proxy hedge is realized with ICE Brent. Until July 2016 the score does not rejects at 99% confidence level the null hypothesis that the model fitted on the proxy is similar to the model fitted on the jet fuel. After July 2016 the NIG model fitted on Brent returns has lost massively its forecasting capacity and became inappropriate.

6. Conclusions

This paper explores the topic of proxy hedging in middle distillates market with a focus on jet fuel. The research addresses the problem of a refinery or an airline company that hedges its jet fuel price risk with proxy instruments including Brent futures and gasoil futures. The problem is studied in two steps : first the various econometric models with fat tails and volatility clustering are explored in relation with the returns of daily time series and second the proxy hedging is test based on density forecasts methods.

The results from the first part show that NIG distribution, APARCH specifications

FIGURE 6: Evolution of the Gneitting Test Score for NIG model and ICE LS Gasoil

of the volatility dynamics capture in an appropriate manner the behavior of jet fuel, brent and gasoil prices. Also GARCH switching regimes model are a good candidate for modeling the markets that might exhibit thin liquidity.

The second part show that the NIG model fitted on the Singapore Gasoil as proxy has the best density forecasting abilities from the considered choices.

A future direction for our research is the consideration of transaction costs in the Gneiting Test score function, as trading future contracts usually involves brokerage commisions and liquidity across different product maturities. This leads to adressing the problem of dimensionality, as it would be necessary to consider a technique such as approximate dynamic programming to produce a hedging policy that reflects such costs.

FIGURE 7: Evolution of the Gneitting Test Score for NIG model with ICE Brent

Annexe

This brief review of the Generalized Hyperbolic distribution functions focuses on the Normal Inverse Gaussian function. The generic form of a Generalized Hyperbolic model is :

$$f(x;\lambda;\chi;\psi;\mu;\sigma;\gamma)=\frac{(\sqrt{\psi\chi})^{-\lambda}\psi^{\lambda}(\psi+\frac{\gamma^2}{\sigma^2})^{0.5-\lambda}}{\sqrt{2\pi}\sigma K_{\lambda}(\sqrt{\psi\chi})}\times\frac{K_{\lambda-0.5}(\sqrt{(\chi+\frac{(x-\mu)^2}{\sigma^2})(\psi+\frac{\gamma^2}{\sigma^2})})e^{\frac{\gamma(x-\mu)}{\sigma^2}}}{(\sqrt{(\chi+\frac{(x-\mu)^2}{\sigma^2})(\psi+\frac{\gamma^2}{\sigma^2})})^{\lambda-0.5}},$$

where $K_{\lambda}(x)$ is the modified Bessel function of the third kind :

$$K_{\lambda}(x)=\frac{1}{2}\int_{0}^{\infty}y^{\lambda-1}e^{-\frac{x}{2}(y+y^{-1})}dy. \tag{21}$$

With properly chosen parameters, this distribution reduces to the following distributions :

1. $\lambda=1$: hyperbolic distribution
2. $\lambda=-1/2$: NIG distribution
3. $\lambda=1$ and $\xi\to 0$: Normal distribution
4. $\lambda=1$ and $\xi\to 1$: Symmetric and asymmetric Laplace distribution
5. $\lambda=1$ and $\chi\to\pm\xi$: Inverse Gaussian distribution
6. $\lambda=1$ and $|\chi|\to 1$: Exponential distribution
7. $-\infty<\lambda<-2$: Asymmetric Student
8. $-\infty<\lambda<-2$ and $\beta=0$: Symmetric Student
9. $\gamma=0$ and $0<\lambda<\infty$: Asymmetric Normal Gamma distribution

Among the Generalized Hyperbolic family, the Normal Inverse Gaussian distribution can be obtained by setting $\lambda=-\frac{1}{2}$ in the previous equation. Thus :

$$f(x;-\frac{1}{2};\chi;\psi;\mu;\sigma;\gamma)=\frac{\chi^{\frac{1}{2}}(\psi+\frac{\gamma^2}{\sigma^2})}{\pi\sigma e^{\sqrt{-\psi\chi}}}\times\frac{K_{1}(\sqrt{(\chi+\frac{(x-\mu)^2}{\sigma^2})(\psi+\frac{\gamma^2}{\sigma^2})})e^{\frac{\gamma(x-\mu)}{\sigma^2}}}{(\sqrt{(\chi+\frac{(x-\mu)^2}{\sigma^2})(\psi+\frac{\gamma^2}{\sigma^2})})}.$$

By changing the variables of the previous equation $c = \frac{1}{\sigma^2}$; $\beta = \frac{\gamma}{\sigma^2}$; $\delta = \sqrt{\frac{\chi}{c}}$; $\alpha = \sqrt{\frac{\psi}{\sigma^2} + \beta^2}$ we obtain a more popular representation, and the density of the $NIG(\alpha,\beta,\mu,\delta)$ distribution is equal to :

$$f_{NIG}(x;\alpha;\beta;\mu;\delta) = \frac{\delta\alpha \cdot exp(\delta\gamma + \beta(x-\mu))}{\pi \cdot \sqrt{\delta^2 + (x-\mu)^2}} K_1(\alpha\sqrt{\delta^2 + (x-\mu)^2}).$$

The moments (mean, variance, skewness and kurtosis) are respectively equal to :

$$\begin{aligned}
\mathtt{E}(X) &= \mu + \delta\frac{\beta}{\gamma} \\
\mathtt{V}(X) &= \delta\frac{\alpha^2}{\gamma^3} \\
\mathtt{S}(X) &= 3\frac{\beta}{\alpha \cdot \sqrt{\delta\gamma}} \\
\mathtt{E}(X) &= 3 + 3(1 + 4(\frac{\beta}{\alpha})^2)\frac{1}{\delta\gamma}
\end{aligned}$$

Thus, the NIG distribution allows for behavior characterized by heavy tails and strong asymmetries, depending on the parameters α, β and δ.

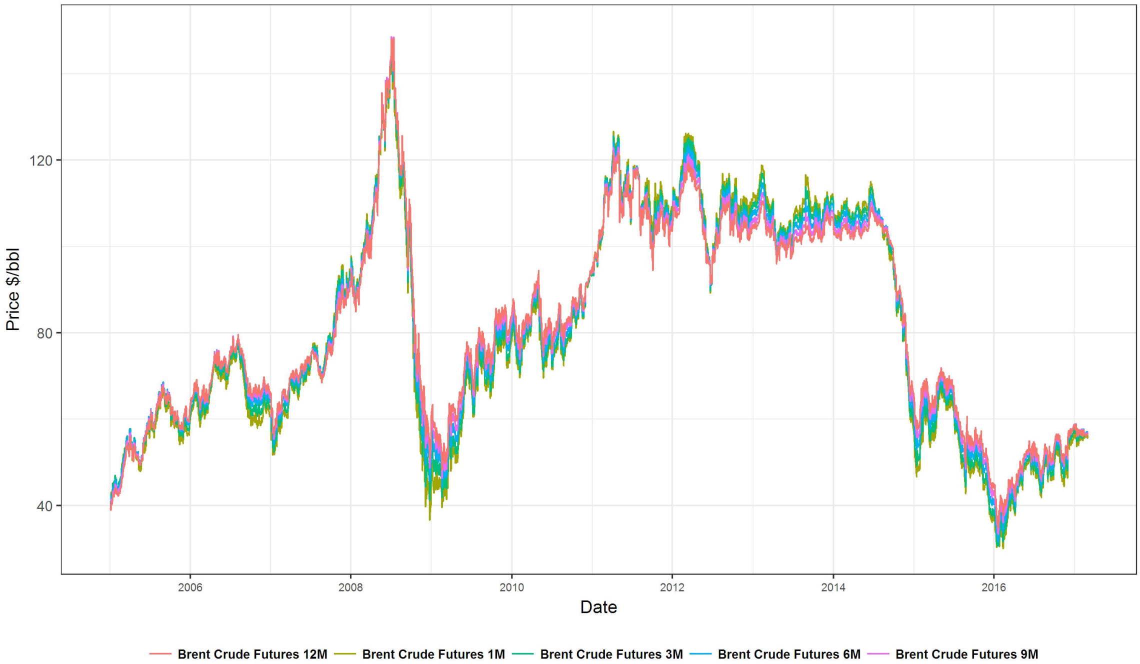

FIGURE 8: ICE Brent Crude Futures

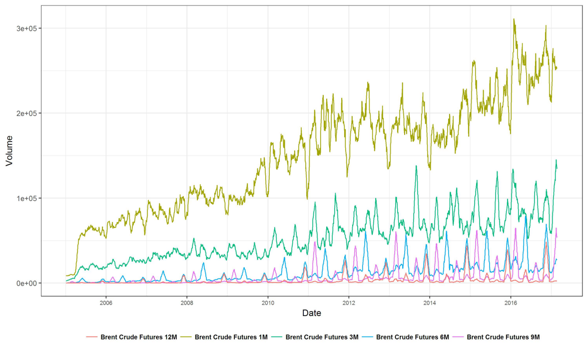

FIGURE 9: Colume Ice Brent Futures

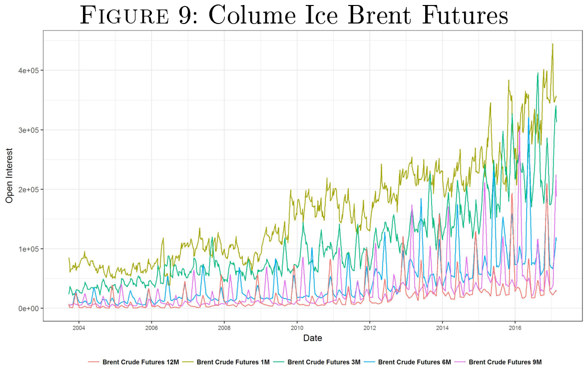

FIGURE 10: Open Interest Ice Brent Futures

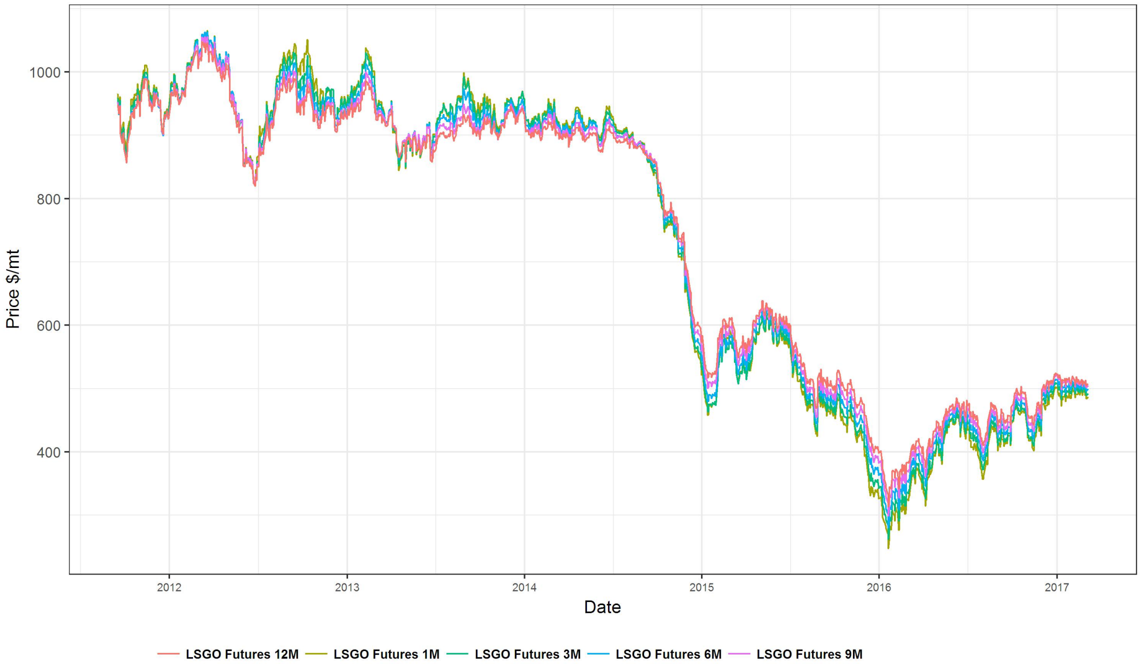

FIGURE 11: ICE Low Sulphur Gasoil Futures

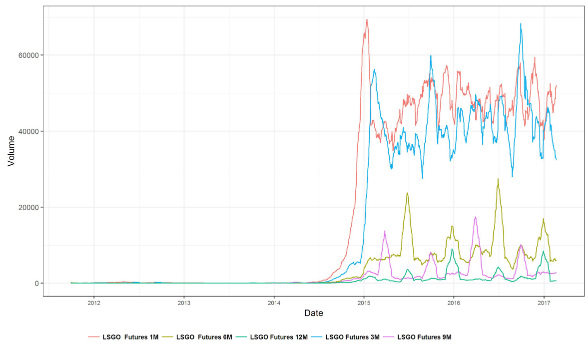

FIGURE 12: Volume ICE Low Sulphur Gasoil Futures Futures

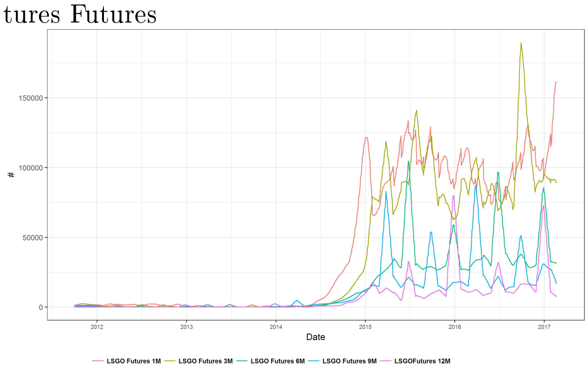

FIGURE 13: Open Interest ICE Low Sulphur Gasoil Futures

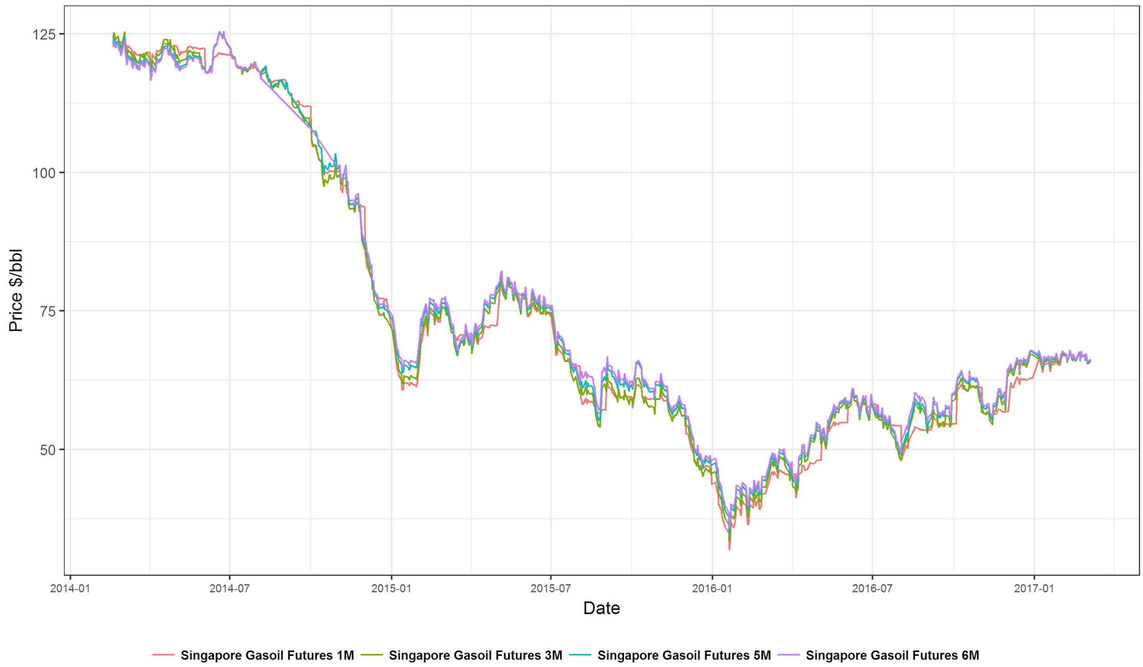

FIGURE 14: Singapore Gasoil 0.5% (Platts) Futures

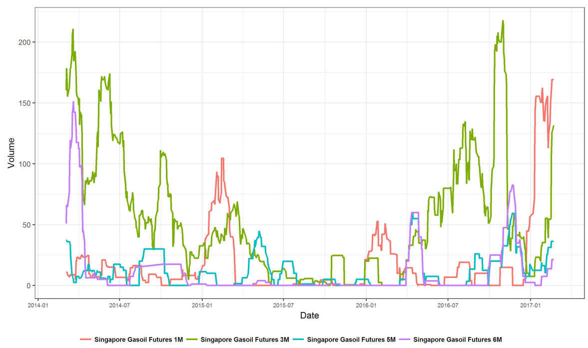

FIGURE 15: Volume Singapore Gasoil 0.5% (Platts) Futures

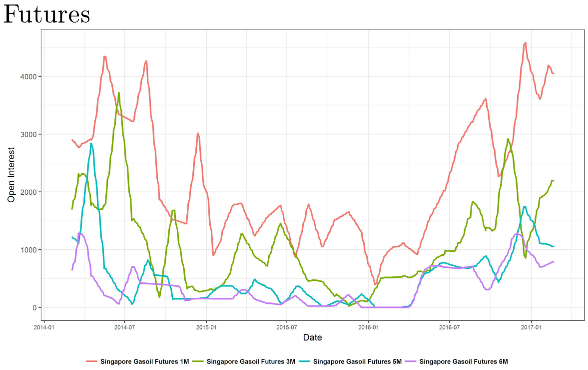

FIGURE 16: Open Interest Singapore Gasoil 0.5% (Platts)Futures

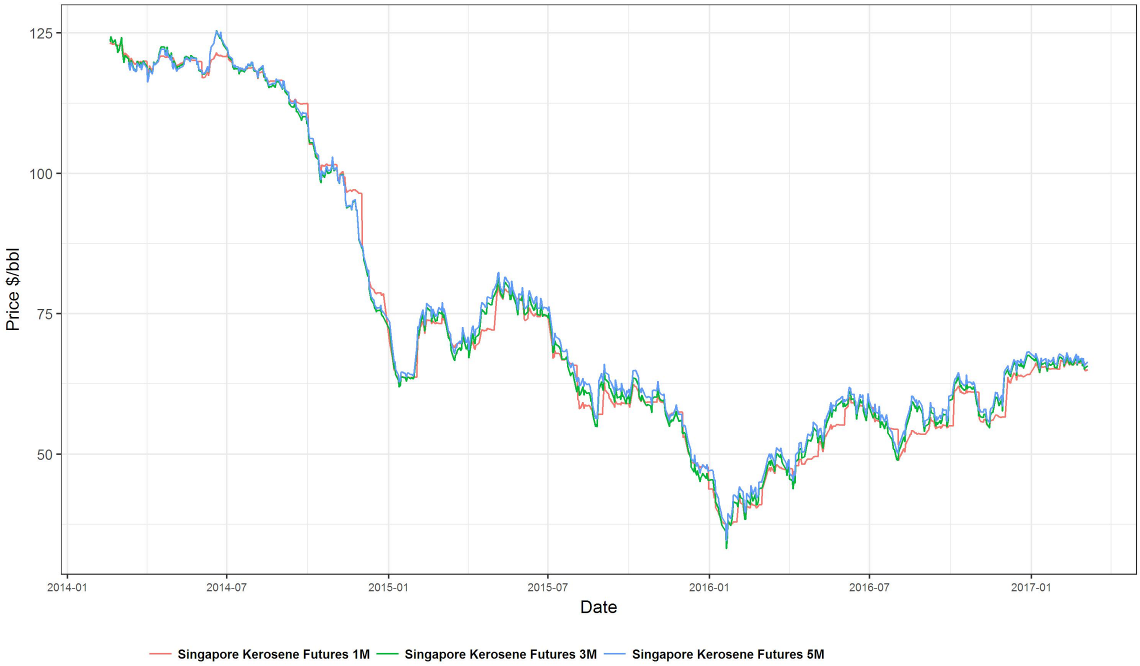

FIGURE 17: Singapore Jet Kerosene (Platts) Futures

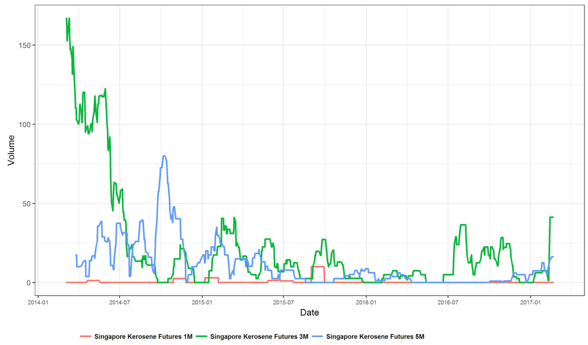

FIGURE 18: Volume Singapore Jet Kerosene (Platts) Futures

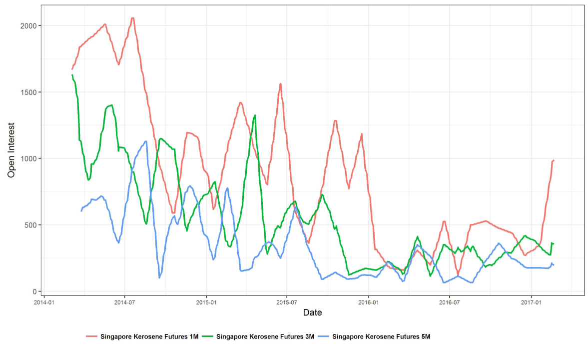

FIGURE 19: Open Interest Singapore Jet Kerosene (Platts)Futures

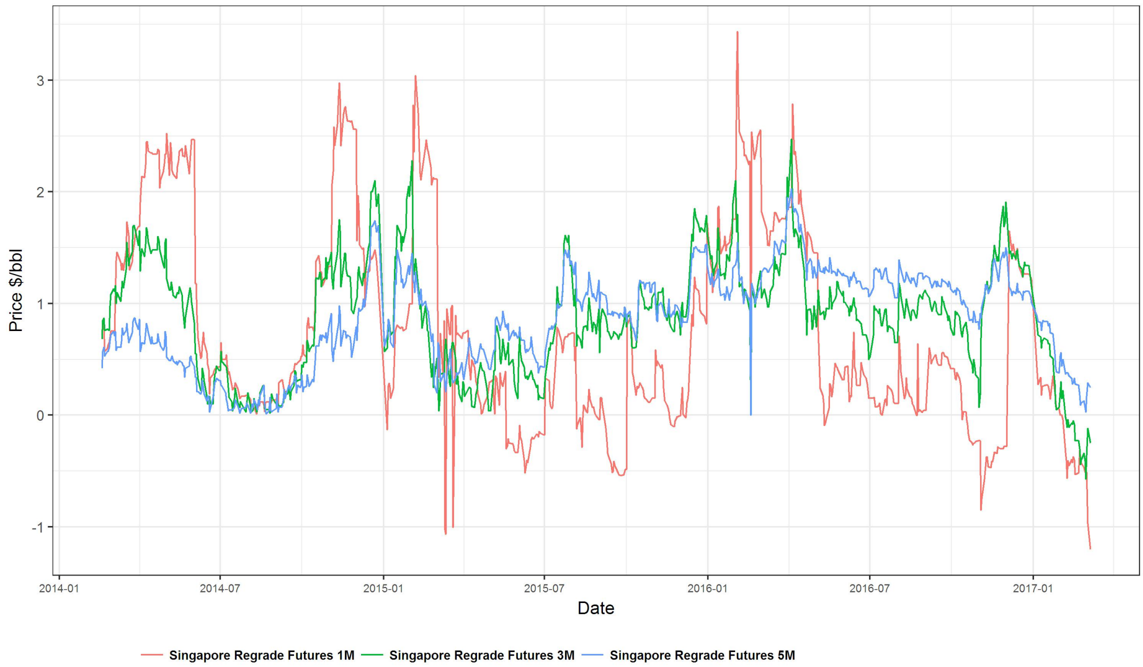

FIGURE 20: Singapore Regrade Futures

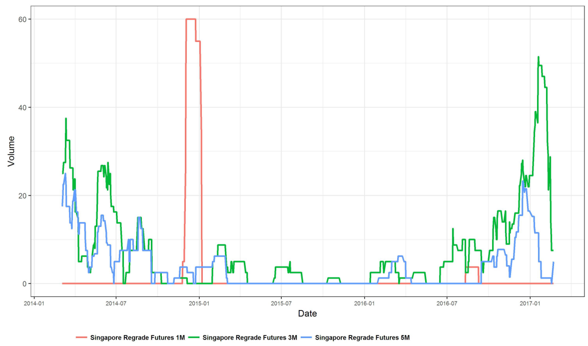

FIGURE 21: Volume Singapore Regrade Futures

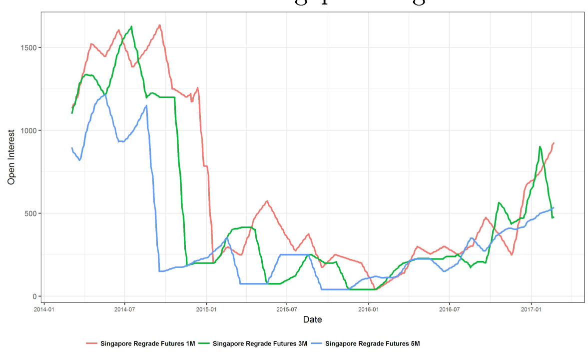

FIGURE 22: Open Interest Singapore Regrade Futures

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