IPO Timing: An Option to Expand*

Hanane Dakhli†

March 29, 2017

Abstract

The existing literature dealing with Initial Public Offering (IPO) shows that IPO generally goes along with the undervaluation of the candidate company and with a significant increase of market prices of listed companies belonging to the same sector before the IPO. Based on this observation, we propose a stochastic model to determine the optimal timing of an IPO by considering this investment decision as a real option to expand. Since investors value the candidates company regarding the market prices of peer firms, IPO may be delayed or withdrown when market prices go down. Our results show that the launch of an IPO can be viewed as the exercise of an expensive real option. Moreover, we prove that the optimal exercise of this option perfectly coincides with the optimal timing for IPOs when there is a positive market trend. Finally, our conclusions also lead us to explain the clustering of IPOs.

Keywords: IPO, Market Timing, Real Option, Clustering.
JEL Classification: G12, G13.

*We are grateful to the participants to the XXXII$^{\text{nd}}$ Conference of the French Finance Association (Cergy, June 2015), the XXX$^{\text{th}}$ Annual Congress of European Economic Association (Mannheim, August 2015), the IX$^{\text{th}}$ Intl. Conference on Computational and Financial Econometrics (London, December 2015) and the XII$^{\text{th}}$ Intl. Conference on Applied Business and Economics (Nanterre, September 2016) for valuable comments.

†Groupe ESC Troyes en Champagne; e-mail: hanane.dakhli@get-mail.fr

Introduction

The initial underpricing during an IPO is one of the most studied problems in the last decades.[1] This phenomenon can be perceived by a positive difference between the first market price and the IPO price. Ibbotson and Jaffe (1975) were the first to study the factors that induce this variation. The latter was confirmed in the 2000s with the IPOs launched by Internet companies.[2] The first consequence of this massive number of IPOs was the overvaluation of these companies and thus high issuing premiums paid by investors. However, stocks held by short-term investors experienced a larger drop. IPOs take place after an exceptional increase in the market index of the companies belonging to the same sector. Lerner (1994) studied IPO decisions taken by venture capitalists and showed that 60 days before the offer, the index of the sector increased by 9.9%. Also, according to Loughran and Ritter (2002), the year preceding IPOs, the candidate companies predict an average expected return of 72%. The first half of this return is related to the rise in prices of peer companies and the second is related to the fact that the market underperform the candidate companies. In addition of these studies, Ritter and Welch (2002) and Gajewski and Gresse (2006), suggest that on average the long-term performance is lower to the short-run one. Consequently, the candidate company tends to exploit his informational advantage in order to choose the optimal timing generally during a temporary window of opportunity (Ritter, 1991; Jacquillat, 1994).

The determination of this favorable opportunity by a private company supposes two implicit assumptions. The first one is that companies have the right to decide the timing of their IPOs. The second is that potential investors use public information related to the sector to value the candidate company. Based on these assumptions, this article has two objectives. First, it consists in developing a dynamic model to define the optimal timing of an IPO.[3] This article tries to determine the IPO timing as regards the flexibility of this decision and the market conditions. The second goal is the use of IPO optimal timing of a particular company in order to explain the clustering of IPOs (Chen and Ritter, 2000; Hansen, 2001).

Following Pástor and Veronesi (2005), the decision of an IPO will be

[1] We can refer to MacDonald and Jacquillat (1974); Ibbotson and Jaffe (1975); Jog and Riding (1987); Ritter (1991); Gajewski and Gresse (2006).

[2] See, for instance, Degeorge and Derrien (2001); Ljungqvist and Wilhelm (2002); Ritter and Welch (2002).

[3] The disclosure between the candidate company and investors is not taken into account.

thus modeled as a real option. The entrepreneur of the company candidate can decide at any time to go public, and thus the exercise of the option of going public. Potential investors determine the value of the company based on publicly and specific available information related to the candidate company. In an efficient market, all the public information is supposed to be incorporated in the price of listed companies similar to the candidate company. Furthermore, the proceeds of the IPO and the value of the candidate depend on the market conditions. We will suppose that information evolves randomly over time. Moreover, uncertainty impacts the value of the option of going public. Indeed, the entrepreneur can always wait for a positive evolution of market prices before going public.

Although we suppose implicitly that the decision of an IPO is based on the evolution of prices of peer companies, other factors may likely lead to it. The decision of an IPO can be justified by reasons related to the candidate company and gathering all the factors in the same model is difficult to consider. For this reason, the existing models do not seem to take into account all the motivations of an IPO. We can quote the diversification of shareholders portfolios (Leland and Pyle, 1977), the financing of new projects (Chemmanur and Fulghieri, 1999; Kim and Weisbach, 2005), the development in the capacity of negotiation (Rajan, 1992), a better governance (Pagano and Röell, 1998), the appreciation of the notoriety of the company (Merton, 1987) and the preparation in the change of control for a possible transfer of the company (Zingales, 1995; Mello and Parsons, 1998).[4] Without all these strategic motivations, the decision of an IPO can be the difference between the value of the company before and after the IPO (Ritter, 1991). The company prior to the IPO is supposed to have a cost of capital higher than the market and therefore a lower value (Jacquillat, 1994). Burgstaller (2009) suggests that companies go public after an increase of the market index in order to benefit from the low costs of capital. Thus, the decision of an IPO is made when the value of the listed company with regards to the costs of the offer and its value prior to the IPO.

While trying to identify the most important factors to take into account during the procedure of an IPO and the choice of its timing, Brau and Fawcett (2006) and Ritter and Welch (2002) show that market condition is the most accurate factor that influences the timing of going public. According to Burgstaller (2009), IPOs do not depend on the business cycle of the candidate companies but on market prices of the listed companies. Compa-

[4]Other motivations were also quoted by Benninga *et al.* (2005).

nies tend more to go public when market prices are high. This also implies that candidate companies choose their IPO timing in order to take advantage from optimistic investors. During these periods, the market tends to overestimate the candidate company by reducing the cost of capital. Pagano *et al.* (1998) showed that the rise of the market index has a significant impact on the probability of going public of the companies from the same sector, and investors interpret the temporary increase of returns as a significant long-term indicator.

Before the IPO, the entrepreneur of the candidate company receives dividends and holds the option of going public. An IPO implies the exercise of this option and thus the loss of its value. The value of this option must thus be considered as one of the costs of the offer. Under these conditions, the IPO is launched when all the costs can be covered at least by the difference between the value of the candidate before and after the IPO.

The value of the going public option and market conditions are the main constraints to be taken into account in the model. The entrepreneur admits that the fluctuation of the market prices is random and waits for a bull period before going public. The exercise of the IPO option can thus be optimal only after exceptional increases in the market index. Thus, the value of the option increases with the volatility of the market index. If the company is not ready for the IPO, the entrepreneur have to wait because of the value of the IPO option. According to Latham and Braun (2010), the most important risk incurred by a company during its IPO corresponds to external conditions, out of the control of the entrepreneur. Then, the lack of adequate preparation or the choice of a bad timing can result in the cancellation of the offer or its failure. Consequently, the increasing number of IPOs during market peaks is related to the wait of a bull market. This result is not related to the exploitation of the entrepreneur of his informational advantage, but to the stochastic evolution of market index.

In addition, an externality related to information has an important role when determining the optimal timing of an IPO.[5] It impacts the relation between IPO clustering and initial underpricing. Consequently, under the assumption where the market index is used to value the candidate company, this externality becomes an important factor influencing our model. We suppose then that the information related to the candidate company and its IPO is exogenous and the decision will be studied conditionally to market

[5] See Subrahmanyam and Titman (1999) and Benveniste *et al.* (2002).

condition.

This article is organized as follows: the first section presents the model. The second section examines, first, the problem of the IPO optimal timing. The problem of maximization of the entrepreneur is subsequently defined. Secondly, the value of the IPO option and the value of the company are formalized. Thirdly, a discussion of the results of the model is presented *via* an empirical study based on a sample of companies listed on the French Stock market between 2005 and 2012. The last section concludes.

1 The Model Presentation

We consider at $t = 0$, a risk averse entrepreneur. He provides an initial investment I_0 at the inception of the company. The business generates at t a profit π_t that evolves continuously over time, with an initial profit π_0. The value of the company is computed as the present value of its future profit flows. At any time after $t = 0$, the entrepreneur can sell the totality or part of its shares through an IPO. There are no new costs related to new investments, others that the costs of exploitation.

The potential offer of shares is mainly motivated by financing needs.[6] The proportion $0 < x < 1$ of shares to be traded is determined exogenously. The choice of the proportion x can also be made endogenously. Indeed, according to Leland and Pyle (1977), the entrepreneur of the company emits a signal to specify the quality of its company through the proportion of the shares retained after the offer. Moreover, for a given level of risk, the entrepreneur receives higher profits for a "good quality" than a "bad quality" company. The retained proportion can also be impacted by the degree of aversion to the risk of the entrepreneur. A risk averse entrepreneur will retain a small proportion of the shares, which increases x. This choice can also be influenced by a control motivation. Zingales (1995) argues that the sale of a strategic proportion of shares during an IPO is justified by the realization of higher profits. Thus, the issue of shares to external investors does not affect the profit flows of the company. In addition, the decision of an IPO in the model is determined by the comparison of the value of the

[6] Ritter and Welch (2002) describe the main motivation for the majority of IPOs like the desire of raising capital for the company and to create a Common Market in which the founders of the company and the other shareholders can convert part of their wealth in cash. Kim and Weisbach (2005) studied nearly 17,000 IPOs in 38 countries and noted that 79% of the companies listed raise capital through this public offering.

company prior the IPO and the value of the company after the IPO and its proceeds. The two values are given based on their respective profit flows, their uncertainty and the cost of capital employed by the entrepreneur and the market.

1.1 The Impact of Information

The valuation of the company requires the determination of its future profit flows. Prior to the IPO, investors cannot observe its profits. Only the entrepreneur knows π_t. These profit flows depend on two factors: the characteristics of the candidate company and other external factors. Consequently, companies that belong to the same sector are affected by the same external factors and their profits are then correlated to the profit of the company candidate. Thus, through the examination of the profits of the companies belonging to the same sector, potential investors obtain preliminary information related to level of profit of the candidate company. In order to limit the impact of the profit of a particular company, a sectorial average profit $\overline{\pi}_t$ is considered.

Private information that corresponds to the quality of the company, remains stable. The candidate company can be of either good or bad quality. The level of profit at any time t will thus be the product of the sectorial average profit and a constant relating to the quality of the company (b or g). Thus, we have:

$$\begin{cases} \pi_t = b\overline{\pi}_t \\ \pi_t = g\overline{\pi}_t, \end{cases} \tag{1}$$

where:

$$0 < b < 1 < g < \infty, \tag{2}$$

$$\frac{b+g}{2} = 1. \tag{3}$$

The profits of the company are perfectly correlated with the profit of the sector. This assumption can be released in order to allow the variation of the profit of the company according the time. The level of profit of the company then becomes less perfectly correlated but systematically distributed around the sectorial average profit. Thus, we suppose that the sector is made of an equal number of companies of "good and bad quality" in order to determine $\overline{\pi}_t$.

When the company decides to go public, the entrepreneur must provide all the financial information related to the company in order to inform potential investors. This information corresponds to a signal enabling them to confirm and update their information regarding the company and to know its current profit level. In order to eliminate any potential strategic interaction between the entrepreneur and the investors and following Allen and Faulhaber (1989), we suppose that this signal is perfectly revealing the real quality of the company. Thus the optimal timing of the IPO becomes a problem of decision of the entrepreneur.

During the valuation of the company, the entrepreneur and the investors have the same uncertainties in regards to the future profit flows of the candidate company. We suppose thus that the profits of the company and the market index increase with at instantaneous growth rate μ. The current profits will be also impacted by the random changes of the market. Indeed, any unexpected change of the external factors will be observable publicly and will impact the profits of the companies listed or not, in equal proportions. We suppose that the profit of the sector follows a geometric Brownian motion. The stochastic differential equation of the profit of the sector corresponds then to:

$$\frac{d\overline{\pi}_t}{\overline{\pi}_t} = \mu dt + \sigma dz, \tag{4}$$

where z is a standard Brownian motion.

The equation (4) is valid for all types of companies. This can be checked through substitution of π_t by $b\overline{\pi}_t$ or $g\overline{\pi}_t$. Consequently, using π_t, it is possible to determine the optimal timing of the IPO. Indeed, conditionally on the level of the initial profit π_0, the investors can at any time determine the level of profit of the company candidate π_t by using the equation (4) between 0 and t. When $\sigma = 0$, the level of the profit at t will be equal to:

$$\pi_t = \pi_0 e^{\mu t}. \tag{5}$$

With uncertainty, the conditional expected profit corresponds to:

$$E\left[\pi_t \left| \pi_0 \right.\right] = \pi_0 e^{\mu t}. \tag{6}$$

The impact of new information, implicit in σdz, will depend on the value of σ. Contrary to mature sectors, new information relative to emergent sectors will have a more significant impact on the market values. This distinction would be relevant when forecasting the IPO optimal.

1.2 The Relation between Risk and Value

The market is composed of investors having the same level of aversion to risk and who have access to same information. They adopt well diversified strategies. Thus, to value an asset, they discount the future cash-flows at an adjusted appropriate rate r^m, specified exogenously.[7] The entrepreneur invests his wealth in the candidate company and he does not have access to any financing from the financial markets. He thus supports the cost of the idiosyncratic risk. The entrepreneur is constrained to discount the future profit flows of his company at a rate $r^e > r^m > \mu > r^f$ where r^f is the risk free interest rate (Eckbo and Norli, 2005). According to Sahlman (1990), venture capitalist use a discount rate between 40% and 60% for startups and between 25% and 33% for listed companies. By using the CAPM, Merton (1987) showed that the increase in the number of investor-shareholders of the company reduces the cost of capital and increases the value of market of the company. Moreover, IPO improve the borrowing capacity and increase the company's value. A high level of specific risk of the company increases significantly its cost of capital (Merton, 1987). Moreover, Mauer and Senbet (1992) suggest that the combination of a partial hedging and a restricted number of investors decreases the value of the company prior the IPO.

All the profits will be distributed as dividends to the shareholders. The present value of the company can then be given by using the discount rates r^e and r^m. Consequently, conditional on π_t, we can determine the value of the candidate company. With $w = Ln(\pi_t)$, we determine the stochastic differential equation of w which follows a geometrical Brownian motion, by using the Itô integral:

$$dw = \left(\mu - \frac{1}{2}\sigma^2\right) dt + \sigma dz.$$

The determination of $E[\pi_t]$ is then equivalent to the calculation of:

$$E\left[e^w \left| w_o = w\right.\right] = e^{w\left[\left(\mu - \frac{1}{2}\sigma^2\right) + \frac{1}{2}\sigma^2\right]}.$$

The multiplication of the preceding expression by e^{-rt} and the replacement of e^w by π_t, enable us to obtain:

$$V^i(\pi_t) = E\left[\int_t^\infty \pi_a e^{-r^i(a-t)} da \middle| \pi_t\right] = \frac{\pi_t}{r^i - \mu}. \tag{7}$$

[7] See Hart and Moore (1995)

With $\Delta^i = r^i - \mu$, the equation (7) becomes then:

$$V^i(\pi_t) = \frac{\pi_t}{\Delta^i}. \tag{8}$$

Δ^i is equivalent to an implicit dividend resulting from the investment in the company. The market equilibrium requires that the return of the investment in the company, the profit flow and the capital gain be equal to the required return of the market r^m. In order to solve the problem of the optimal timing of the IPO, the entrepreneur proceeds to a comparison between the market value of the company and its own evaluation. The market value can be computed as follows:

$$V^m(\pi_t) = \frac{\pi_t}{\Delta^m}. \tag{9}$$

The value of the candidate company computed by the entrepreneur corresponds to:

$$V^d(\pi_t) = \frac{\pi_t}{\Delta^e}. \tag{10}$$

The market value of the company will always be higher than that the value determined by the entrepreneur, since $\Delta^m < \Delta^e$.[8] The value of the company is given based on its profit flows. Thus by applying the Itô lemma, we obtain:

$$dV^i(\pi_t) = V^{t'} d\pi_t + \frac{1}{2} V^{i''} (d\pi_t)^2 \tag{11}$$

$$= \frac{1}{\Delta^i} (\mu \pi_t dt + \sigma \pi_t dz). \tag{12}$$

By the substitution of $\pi_t = V^i \Delta^i$, we obtain:

$$\frac{dV^i}{V^i} = \mu dt + \sigma dz. \tag{13}$$

Consequently, the value of the company, for the entrepreneur and the investors, has the same stochastic properties as the profit flows generated by the company.

The uncertainty of future profits increases with the change in market conditions. The fixed values of g and b exclude the specific changes of the

[8] According to Pratt (2008), this trade-off, adjusted with the changes of the index prices between 1975 and 1985, vary between 60% and 80%. Blackwell and Pavlik (1999) also obtained similar results for IPO between 1989 and 1990 (see also Pavlik and Dare, 2002). The average trade-off between the shares prices before and after the IPO is 75%.

company. The uncertainty of the market conditions, represented by σdz can be broken up into two parts: the systemic and specific risks. Let us note that although the investors diversify the idiosyncratic risk and value only the systemic risk, the entrepreneur must support both two risks.

2 Optimal Timing Decision

The entrepreneur is the only decision maker on the IPO. Waiting for the optimal timing makes his decision an optimal stopping problem. The decision of going public is irreversible. However it is important to note that this constraint does not exist in practice. Indeed, since the freezout option has a relatively low value, it does not affect the decision of the IPO optimal timing.

This assumption can be justified for several reasons. First, the number of freezouts is relatively weak compared to IPOs. Kaplan (1991) studies 183 LBOs for a value of more than 100 million dollars between 1979 and 1986. On the same period, Loughran and Ritter (1995) enumerated 2,683 IPOs. The ratio of Freezouts/IPOs is roughly equal to 7%. Moreover, the majority of these freezouts were initiated by non-managing shareholders. The number of freezouts initiated by managers to go back private is weak, which suggest that the option of freezout when deciding to go public has a low value. In addition, according to Mikkelson *et al.* (1997), the proportion of the capital retained by the entrepreneur decreases during five to ten years after the IPO, which suggests that the insiders are more interested by the diversification of their portfolios than by repurchasing their companies. Lastly, a significant percentage of the listed companies change their control structure during the five years following the IPO. Consequently, the IPO optimal timing will be formalized under the assumption that the decision of IPO and the IPO can be realized instantaneously.

The decision of going public is made towards the comparison between the proceeds of the offer and the value of the company prior to the IPO. The optimal timing can be found when the offer increases the wealth of the entrepreneur. Institutional constraints make this decision complex. The company must satisfy certain conditions before the IPO. Indeed, the maturity of the company and a sufficient historical profit and sales turnover are constraints that can imply the delay of the offer. These institutional constraints will not be taken into account in our model.

In order to determine the IPO optimal timing, a procedure in two stages will be followed. The optimal timing of IPO is first given for the deterministic case. Then, the calculation of the value of the option of IPO and the value of the company will be presented based on stochastic profit flows. However, before the problem of maximization of the entrepreneur will be defined.

2.1 The Entrepreneur Decision Problem

The entrepreneur choose the IPO optimal timing in order to maximize the present value of the future profits of its x shares. Prior to the IPO the profits basically come from dividends and after the IPO from the proceeds of the offer.

During the IPO, the entrepreneur supports the costs of the offer (Ritter, 1987). These costs can take two forms: direct expenditure and issuing costs. The direct expenditure takes into account the IPO prospectus expenses, the legal and other administrative expenditure. The expenses of issuing correspond to a percentage of the value of the IPO proceeds. Lee *et al.* (1996) consider all the costs combined as a percentage of the IPO proceeds. They evaluated the costs approximately at 17% for the small offers and 6% for important offers. These two costs are taken into account in the trade-off γ between the IPO price and the price paid by the investors where $0 < \gamma < 1$ and C the fixed direct costs. Moreover, an additional component at the cost C corresponds to information costs. These costs can be compensated by the initial underpricing (Chemmanur and Fulghieri, 1999).

The net proceeds of the IPO $\mathrm{P}(\pi_t)$ correspond then to:

$$\mathrm{P}(\pi_t) = x\frac{\pi_t}{\Delta^m}(1-\gamma) - C. \tag{14}$$

Investors receive the proportion γ of the proceeds of the IPO, while leaving for the entrepreneur $(1-\gamma)$. In addition, the value of x shares held by the entrepreneur, or the value of the private company $\Sigma(\pi_t)$, is composed by the dividend flows and the proceeds of the IPO:

$$\Sigma(\pi_t) = E\left[\int_t^{t+T(\pi^*)} x\pi_a e^{-r^e(a-t)}da + e^{-r^e T(\pi^*)}\mathrm{P}(\pi^*)\,|\pi_t\right]. \tag{15}$$

The first term of the equation (15) corresponds to the present value of the dividend flows cumulated by the entrepreneur before the IPO. The second term is the present value of the IPO proceeds. The objective of the entrepreneur is to choose a strategy enabling him to determine the optimal timing of IPO that maximizes $\Sigma(\pi_t)$. The level of profit for which the company goes public corresponds to π^*. $T(\pi^*)$ is the first time where the stochastic process of π_t reaches π^*, with $T(\pi^*)$ a random variable given initial information.

When $T(\pi^*) > dt > 0$, one waiting period is necessary before the IPO. Consequently, $\mathrm{P}(\pi_t)$ will only depend on parameters related to the step preceding the offer. $\Sigma(\pi_t)$ can thus be given in two times: the dividend immediately received and the present value of the shares of the company before the offer. The equation (15) can then be rewritten as follows:

$$\Sigma(\pi_t) = x\pi_t dt + \frac{1}{1+r^e dt} E\left[\Sigma(\pi_t + d\pi_t)\,|\pi_t\right]. \tag{16}$$

When $T(\pi^*) = 0$, the entrepreneur decides to go public. Then:

$$\Sigma(\pi_t) = \mathrm{P}(\pi_t). \tag{17}$$

With uncertainty, the entrepreneur is unable to specify the IPO optimal timing *ex-ante*. Thus the determination of the IPO timing consists in finding the critical level of profit for which the proceeds of the offer are at least equal to the value of the company prior the IPO. Since the entrepreneur is the only decider of the IPO, he will choose between keeping its company private and receiving the value determined in (16) or to going public and receiving the value determined by (14) as follow

$$\begin{aligned}\Sigma(\pi_t) = & Max\left\{x\frac{\pi_t}{\Delta^m}(1-\gamma) - C; x\pi_t dt\right.\\ & \left. + \frac{1}{1+r^e dt}E\left[\Sigma(\pi_t + d\pi_t)\,|\pi_t\right]\right\}.\end{aligned} \tag{18}$$

2.2 Determinist Profit Flows

As a first step in the resolution of the IPO optimal timing problem, we consider a naive or myopic entrepreneur. He decides to go public as soon as the proceeds of the offer are equal to the value of its shares before the IPO. The optimal timing of the IPO corresponds to the present value of the company, for a null critical level of profit. It can be obtained by equalizing the equation (14) with the value of the company before the IPO:

$$x\frac{\pi^n}{\Delta^m}(1-\gamma) - C = x\frac{\pi^n}{\Delta^e}. \tag{19}$$

The resolution of the equation (19) enables us to obtain the critical level of profit for a myopic entrepreneur π^n:

$$\pi^n = \frac{C}{x\left(\frac{1-\gamma}{\Delta^m} - \frac{1}{\Delta^e}\right)}. \tag{20}$$

With, for any positive π^n:

$$\frac{\Delta^m}{(1-\gamma)\,\Delta^e} < 1. \tag{21}$$

It is a necessary condition in order to make the market value of the company higher after the IPO and take into account the trade-off between the IPO price and the issuing price. This condition guarantees that the trade-off the market value of the company is a monotonously increasing function of the level of profit. This condition also ensures the existence of a single solution to the stopping problem.

A second assumption is imposed on the values of the parameters. Indeed, the critical level of profit π^n must be higher than the initial profit π^0. This assumption guarantees that the company will not go public immediately after its inception.

For deterministic porfits, the entrepreneur knows the level of the profit π_{t+T} regarding the profit at T. We suppose that the IPO occurs at $t+T$. The present value of the cumulated profit flows can thus be found from the equation (15). By substituting in equation (15), π_t by π_a and $\mathrm{P}(\pi_t)$ by its expression, we obtain:

$$\Sigma(\pi_t) = \int_t^{t+T} x\pi_t e^{\mu(a-t)} e^{-r^e(a-t)} da + e^{-r^e T}\left[x\frac{\pi_t e^{\mu T}}{\Delta^m}(1-\gamma) - C\right]. \tag{22}$$

The IPO is planned to maximize expression (22). To find the level of profit for which the offer will take place, as regards the deterministic level of profit, the expression (22) is maximized, for T. Then we have

$$\Sigma(\pi_t) = x\frac{\pi_t}{\Delta^e}\left(1 - e^{-\Delta^e T}\right) + x\frac{\pi_t e^{-\Delta^e T}}{\Delta^m}(1-\gamma) - Ce^{-r^e T}. \tag{23}$$

The maximization of $\Sigma(\pi_t)$ for T, enables us to have the following first order condition:

$$\Sigma_T(\pi_t) = \Delta^e x\frac{\pi_t}{\Delta^e}e^{-\Delta^e T} - \Delta^d x\frac{\pi_t e^{-\Delta^e T}}{\Delta^m}(1-\gamma) + r^e Ce^{-r^e T}. \tag{24}$$

Setting equation (24) equal to zeor we obtain the optimal period of waiting before the IPO T^*

$$T^* = \frac{1}{\mu} Ln \left[\frac{r^e}{\Delta^e} \frac{C}{x\pi_t \left(\frac{1-\gamma}{\Delta^m} - \frac{1}{\Delta^e} \right)} \right]. \tag{25}$$

The optimal period of waiting cannot be negative. When the difference between the market value of the company and the value before the IPO is higher than C then $T > 0$, If not $T = 0$. [9]

The critical profit allowing the IPO is found at $T^* = 0$. Consequently when the equation (25) is equal to zero, we obtain:

$$\pi^e = \left(\frac{r^e}{r^e - \mu} \right) \frac{C}{x \left(\frac{1-\gamma}{\Delta^m} - \frac{1}{\Delta^e} \right)} = \left(\frac{r^e}{r^e - \mu} \right) \pi^n, \tag{26}$$

with:

$$\frac{r^e}{r^e - \mu} > 1. \tag{27}$$

The expression (26) shows that waiting has a value even for the deterministic case. The entrepreneur must wait before the IPO. This is due to the difference between the costs of the capital with which future flows of profits and the cost of the offer C are discounted.

The value of shares grows at a rate μ. However their future values are obtained *via* discounting at the rate $r^e > \mu$. By delaying the offer, the present value and the cost of the offer C decrease. Thus, the shares lose value, in terms of present value, as regards the important cost related to the IPO.

The critical profit π^e corresponds to the situation where waiting is compensated by the difference between value of the company before and after the IPO. The comparison between $P(\pi^e)$ and the value of the private company shows that the entrepreneur will renounce positive profit resulting from the offer until the IPO, for the level of deterministic profit π^e. Indeed:

$$x \frac{\pi^e}{\Delta^m} (1 - \gamma) - C - x \frac{\pi^e}{\Delta^e} = \frac{\mu C}{r^e - \mu} > 0. \tag{28}$$

Nevertheless it is necessary to interpret the preceding results with precaution. With deterministic flows of profits, there is no uncertainty and the

[9]This can be justified by showing that the second order condition of equation (23) is negative at T. Consequently, T is a maximum.

cost of capital can be only the risk free rate. Consequently $r^e = r^f$. If r^e and r^m converge towards r^f, in the event of certainty, the entrepreneur will not go public since the value of its company will not increase. However, by supposing that r^e and r^m are different from r^f, the problem of decision of the entrepreneur can be easily solved. Moreover, growth rate must also be adjusted, so that the condition $\mu < r^e$ is satisfied.

We define:

$$R^e = \frac{r^e}{r^e - \mu}. \tag{29}$$

The critical level of profit π^e becomes then:

$$\pi^e = R^e \pi^n. \tag{30}$$

With the fixed costs of the offer and without uncertainty, the entrepreneur could wait before the IPO. However, it is not possible to suggest that the entrepreneur exercises only strategically his IPO option. Indeed, without any uncertainty, the entrepreneur knows since the creation of his company the optimal timing of its IPO. Moreover, although the entrepreneur has the IPO option, he will exercise it only when $\pi_t = \pi^e$. Furthermore, with a deterministic evolution of profit flows, it is not possible to affirm that the entrepreneur is able to determine the period of optimal trends.

2.3 Stochastic Profit Flows

When profits are uncertain, the flexibility of the optimal timing decision becomes valuable. Moreover, the possibility of a sequence of positive profits during a short period makes waiting more attractive. Unlike the deterministic case where the entrepreneur knows exactly the timing of the IPO, in case of uncertainty, the timing of the offer is unknown *ex ante*. However, it is possible to determine a critical level of profit for which the optimal IPO is possible.

The objective function of the entrepreneur is the following:

$$\Sigma\left(\pi_t\right) = Max\left\{x\frac{\pi_t}{\Delta^m}\left(1-\gamma\right) - C; x\pi_t dt + \frac{1}{1+r^e dt}E\left[\Sigma\left(\pi_t + d\pi_t\right)|\pi_t\right]\right\}.$$

Time does not have any impact in this analysis. Thus we will remove the index t for more simplicity. The second argument of the objective function can be rewritten as follow:

$$\left(1+r^e dt\right)\Sigma\left(\pi\right) = x\pi\left(1+r^e dt\right) + E\left[\Sigma\left(\pi+d\pi\right) - \Sigma\left(\pi\right) + \Sigma\left(\pi\right)|\pi\right]. \tag{31}$$

By eliminating the second order terms of variation dt^2 we obtain:

$$r^e\Sigma\left(\pi\right)dt = x\pi dt + E\left[d\Sigma\left(\pi\right)|\pi\right]. \tag{32}$$

The equation (32) is an equilibrium condition. The present value of the shares $\Sigma(\pi)$ for a short period dt correspond to the right part of the equality (32). The latter must be equal to the value of the company after its IPO, formulated by the left part of the equality. The total return corresponds to the immediate dividend and expected returns of the shares. The uncertainty of the value of the shares $\Sigma(\pi)$ is caused by the uncertainty of their profit flows. These shares must have an expected market return equal to r^m, since $r^m < r^e$. The entrepreneur is then certain to go public, because he is unable to support the costs of being private. The difference between r^e and r^m corresponds to an opportunity cost. Thus the difference of values $V^m(1-\gamma) - V^e$ grows proportionally to π. Consequently, the increase in the profits involves the increase in the costs of being private.

The development of $d\Sigma(\pi)$, by using the Itô lemma, for equation (32) and canceling the dt terms, enables us to obtain the second order differential equation as follow:

$$\frac{1}{2}\sigma^2\pi^2\Sigma''(\pi) + (r^e - \Delta^e)\pi\Sigma'(\pi) - r^e\Sigma(\pi) + x\pi = 0. \tag{33}$$

To find the critical value of the company in case of uncertainty, the function $\Sigma(\pi)$ that enables us to solve the preceding differential equation must satisfy the following boundary conditions:

$$\Sigma(\pi) = 0, \tag{34}$$

$$\Sigma(\pi^*) = x\frac{\pi^*}{\Delta^m}(1-\gamma) - C, \tag{35}$$

$$\Sigma'(\pi^*) = x\frac{1-\gamma}{\Delta^m}. \tag{36}$$

The first condition (34) indicates that if the value of the company is equal to zero, the value of the option of IPO is null. The conditions (35) and (36) are respectively the value matching and the smooth pasting conditions of the critical value π^*.

Following Dixit and Pindyck (1994), the solution of the second order of the differential equation (33) is:

$$\Sigma(\pi) = A_1\pi^{\beta_1} + A_2\pi^{\beta_2} + \frac{x\pi}{\Delta^e}. \tag{37}$$

The first two terms of equation (37) are the solution of the homogeneous part of the differential equation (33) and the third term corresponds to the integral of the whole differential equation. Replacing $A_1\pi^{\beta_1}$ and $A_2\pi^{\beta_2}$ by their values in the differential equation (33), we obtain the following quadratic equation:

$$\frac{1}{2}\sigma^2\beta(\beta-1)+(r^e-\Delta^e)\beta-r^e=0. \tag{38}$$

The solution of this equation enables us to have the two roots β_1 and β_2.

$$\beta_1=\frac{1}{2}-\frac{r^e-\Delta^e}{\sigma^2}+\sqrt{\left(\frac{r^e-\Delta^e}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2r^e}{\sigma^2}}>1 \tag{39a}$$

$$\beta_2=\frac{1}{2}-\frac{r^e-\Delta^e}{\sigma^2}-\sqrt{\left(\frac{r^e-\Delta^e}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2r^e}{\sigma^2}}<0 \tag{39b}$$

Since β_2 is negative and the first conditions is null, then A_2 is null. If it were not the case, the value of the option would become very high since the profit tends to move towards 0, which corresponds to a violation of one of the boundary conditions. Thus we can express $\Sigma(\pi)$ as:

$$\Sigma(\pi)=A_1\pi^{\beta_1}+\frac{x\pi}{\Delta^e}. \tag{40}$$

Starting from the smooth pasting condition (36), we have:

$$A_1\pi^{\beta_1}=x\left(\frac{\pi^*}{\beta_1}\right)\left(\frac{1-\gamma}{\Delta^m}-\frac{1}{\Delta^e}\right). \tag{41}$$

Substituting the latter under the value matching condition (35), we obtain:

$$\begin{aligned}
&x\left(\frac{\pi^*}{\beta_1}\right)\left(\frac{1-\gamma}{\Delta^m}-\frac{1}{\Delta^e}\right)+\frac{x\pi^*}{\Delta^e}=x\pi^*\frac{1-\gamma}{\Delta^m}-C,\\
&x\pi^*\left(1-\frac{1}{\beta_1}\right)\left(\frac{1-\gamma}{\Delta^m}-\frac{1}{\Delta^e}\right)=C,\\
&\pi^*=\left(\frac{\beta_1}{\beta_1-1}\right)\frac{C}{x\left(\frac{1-\gamma}{\Delta^m}-\frac{1}{\Delta^e}\right)}=\left(\frac{\beta_1}{\beta_1-1}\right)\pi^n.
\end{aligned} \tag{42}$$

The value of A_1 is then

$$A_1=\frac{(\beta_1-1)^{(\beta_1-1)}}{\beta_1^{\beta_1}}\left(\frac{1-\gamma}{\Delta^m}-\frac{1}{\Delta^e}\right)^{\beta_1}\frac{x^{\beta_1}}{C^{(\beta_1-1)}},$$

with:

$$R^s=\frac{\beta_1}{\beta_1-1}, \tag{43}$$

Then

$$\pi^* = R^s \pi^n. \tag{44}$$

The impact of uncertainty on the decision of IPO optimal timing is more clearly explained by examining the extreme case where π tends to move towards 0. By substituting expression (40) in equation (33), we obtain the following quadratic equation:

$$\frac{1}{2}\sigma^2 \beta_1 (\beta_1 - 1) + (r^e - \Delta^e) \beta_1 - r^e = 0. \tag{45}$$

When $\sigma = 0$,

$$\beta_1 = \frac{r^e}{r^e - \Delta^e}. \tag{46}$$

This implies that:

$$R^s = \frac{r^e}{r^e - \mu}. \tag{47}$$

We suppose that r^e tends to move towards r^f in case of certainty, Δ^e a constant, and R^s tends towards R^e. Moreover, for all $\sigma > 0$, $R^s > R^e$ and $\pi^* > \pi^e$, with a stochastic profit flows the entrepreneur has an additional motivation to wait before going public.

Through the IPO, the entrepreneur sacrifices an opportunity of a bull market. To compensate for this loss, he have to receive a profit resulting from the IPO which is definitely higher than the value of its shares before the IPO. Consequently, this corresponds to the exercise price of the option of the optimal timing.

In addition, IPO's optimal timing option implies additional costs. With the costs of the offer, the expenses of subscription and the underpricing, the exercise of this option has an important value and have to be taken into account as an opportunity cost. Consequently, the optimal timing decision is solved by comparing the proceeds resulting from the IPO and the value of the company before the offer. This value is the sum of the values of the shares of the company and the value of the optimal timing option. The later includes the value of waiting in order to reduce the costs of the offer. With a myopic entrepreneur, the company goes public as soon as the proceeds resulting from the offer exceed the present value of the dividend flows. With the timing option, the company goes public when the following condition is satisfied:

$$x \frac{\pi}{\Delta^m} (1 - \gamma) - C - \frac{x\pi}{\Delta^e} - A_1 \pi^{\beta_1} > 0. \tag{48}$$

This inequality remains valid even for the level of profit π^*. Since we supposed, through expression (21) that

$$\frac{\Delta^m}{(1-\gamma)\,\Delta^e} < 1.$$

Thus the market value of the company is higher than it would be if remained private, for any level of positive profit. The delay of the IPO is thus caused by the importance of the fixed costs C and the value of the option of optimal timing. However, we can show that the value of the company if it remain private is lower than its market value. Indeed, based on the value of the option (41):

$$\begin{aligned} A_1\pi^{\beta_1} &= x\left(\frac{\pi}{\beta_1}\right)\left(\frac{1-\gamma}{\Delta^m} - \frac{1}{\Delta^e}\right) \\ &= \frac{1}{\beta_1}\left[x\frac{\pi}{\beta_1}(1-\gamma) - \frac{x\pi}{\Delta^e}\right]. \end{aligned} \tag{49}$$

Since $\beta_1 > 1$, the value of the IPO option is lower than the difference between the market value of the company and the value of the private company. If the entrepreneur did not have to pay C, the company goes public immediately. However, nothing change the fact that the optimal timing option has a significant impact on the realization of the IPO. In order to specify the importance of this effect, a numerical application on the critical value is realized. However, before this, static properties of the parameters related to the critical value will be presented. We will proceed to the determination of the first order conditions of the critical profit π^* (42), comparably to various parameters, all things being equal.

The variations of the proportion of the proceeds of the IPO paid to the intermediary and the fixed costs C comparably to the level of the critical profit are the following:

$$\begin{aligned} \frac{\partial \pi^*}{\partial C} &= \frac{\beta_1}{x\,(\beta_1-1)\left(\frac{1-\gamma}{\Delta^m} - \frac{1}{\Delta^e}\right)} > 0, \\ \frac{\partial \pi^*}{\partial \gamma} &= \frac{\beta_1}{(\beta_1-1)}\frac{\frac{C}{\Delta^m}}{x\left(\frac{1-\gamma}{\Delta^m} - \frac{1}{\Delta^e}\right)^2} > 0. \end{aligned}$$

The relation between the costs of the IPO γ and C and the critical profit increasing. The importance of the IPO costs can thus be considered as a great dissuasion for the IPO. When these costs are high, the value of the company must be sufficiently important in order to support these costs.

Moreover, the variation of the proportion of capital x issued for IPO comparably to the critical profit level is the following:

$$\frac{\partial \pi^*}{\partial x} = -\frac{\beta_1}{(\beta_1 - 1)} \frac{C}{x^2 \left(\frac{1-\gamma}{\Delta^m} - \frac{1}{\Delta^e}\right)^2} < 0.$$

The critical profit level allowing the IPO has a negative relation with the proportion of the capital related to the offer. Indeed, the more the proportion of the capital retained by the entrepreneur is weak the more the level of profit required becomes high. The company goes public only when the value of proportion x exceeds the value of the company before the IPO. The difference between these two values corresponds to an opportunity cost. An increase in x implies the increase of the costs of the private company, for any level of profit. Consequently, through the IPO, the company must sell an important proportion of capital x. If x is partially determined by financing requirement dedicated to new investments, then companies with consequent requirements will be more likely to go public quickly.

In addition, the effect of the variation of the growth rate on the profits of the company before the IPO can be given only through the calculation of the total differential of expression (42):

$$\frac{d\pi^*}{d\mu} = \frac{\partial \pi^*}{\partial \beta_1} \frac{\partial \beta_1}{\partial \mu} + \frac{\partial \pi^*}{\partial \mu}. \tag{50}$$

It results three partial derivatives as follows:

$$\begin{aligned}
\frac{\partial \pi^*}{\partial \beta_1} &= -\frac{C}{(\beta_1 - 1)^2} \frac{1}{x \left(\frac{1-\gamma}{\Delta^m} - \frac{1}{\Delta^e}\right)} < 0, \\
\frac{\partial \beta_1}{\partial \mu} &= \frac{1}{\sigma^2} \left(-1 + \frac{\frac{\mu}{\sigma} + \frac{1}{2}}{\frac{\mu}{\sigma} + \frac{1}{2} + \frac{\sqrt{2r^e}}{\sigma}} \right) < 0, \\
\frac{\partial \pi^*}{\partial \mu} &= -\frac{\beta_1}{(\beta_1 - 1)} \frac{C}{x \left(\frac{1-\gamma}{\Delta^m} - \frac{1}{\Delta^e}\right)} < 0.
\end{aligned} \tag{51}$$

Substituting these three derivatives in equation (50), we obtain an expression having a negative sign. Consequently, the increase in growth rate has a negative impact on the critical profit level.

The increase in growth rate μ can have two different impacts on the critical profit level. Indeed, for a fixed r^i, the increase in growth rate leads to the increase in the value of the company before and after the IPO. However the increase of the market value is faster since $r^m > r^d$ and π/r^i is convex at r^i. This implies the decrease of the critical profit level, when the growth rate of the profit flows increases. Moreover, the increase of μ leads to the decrease of β_1, and consequently the rise of R^s and π^*. Thus the proceeds of going public will be higher and the value of the optimal timing option will increase.

The signs of the relations between the market discount rate and the critical level of profit on the one hand and the entrepreneur discount rate and the critical level of profit on the other hand are:

$$\frac{\partial \pi^*}{\partial r^m} = \frac{\beta_1}{(\beta_1 - 1)} \frac{C \frac{1-\gamma}{\Delta^{m2}}}{x \left(\frac{1-\gamma}{\Delta^m} - \frac{1}{\Delta^e} \right)^2} > 0. \tag{52}$$

Consequently, the increase in the market discount rate leads to the increase in the critical level of profit.

In addition, in order to find the impact of the variation of the entrepreneur discount rate and the critical level of profit, we determine the expression of the total differential of (42):

$$\frac{d\pi^*}{dr^e} = \frac{\partial \pi^*}{\partial \beta_1} \frac{\partial \beta_1}{\partial r^e} + \frac{\partial \pi^*}{\partial r^e}. \tag{53}$$

The three partial derivatives are thus the following:

$$\begin{aligned}
\frac{\partial \pi^*}{\partial \beta_1} &= -\frac{C}{(\beta_1 - 1)^2} \frac{1}{x \left(\frac{1-\gamma}{\Delta^m} - \frac{1}{\Delta^e} \right)} < 0, \\
\frac{\partial \beta_1}{\partial r^e} &= \frac{1}{\sigma^2 \left[\left(\frac{\mu}{\sigma} + \frac{1}{2} \right) + \frac{\sqrt{2 r^e}}{\sigma} \right]} > 0, \\
\frac{\partial \pi^*}{\partial r^e} &= -\frac{\beta_1}{(\beta_1 - 1)} \frac{C \left(\frac{1-\gamma}{\Delta^{m2}} \right)}{x \left(\frac{1-\gamma}{\Delta^m} - \frac{1}{\Delta^e} \right)^2} < 0.
\end{aligned}$$

Substituting these three derivatives in equation (53), we notice that the critical level of profit decrease with the increase of the entrepreneur discount rate before the IPO.

The increase in the two discount rates has the same effects of μ. A high market discount rate reduces the market value of the company and consequently decreases the motivation of the company to go public. The opposite is true when the entrepreneur discount rate before the IPO increases. In this case, the market value of the company increases which implies quick IPOs. In addition, the increase of r^e leads to the increase of β_1 which afterward decreases the critical level of profit. Consequently, the value of the IPO option becomes lower.

We will finally determine the impact of the variation σ of on the critical level of profit. However, we cannot express this relation without using β_1. Consequently, we have:

$$\frac{\partial \pi^*}{\partial \sigma} = \frac{\partial \pi^*}{\partial \beta_1}\frac{\partial \beta_1}{\partial \sigma}.$$

According to equation (51), $\partial \pi^*/\partial \beta_1$ is negative. Thus, to show that $\partial \beta_1/\partial \sigma$ is negative, we will follow Dixit and Pindyck (1994). We define then the (39a) as Z. The total differential of Z for σ gives:

$$\frac{\partial \mathrm{Z}}{\partial \beta_1}\frac{\partial \beta_1}{\partial \sigma} + \frac{\partial \mathrm{Z}}{\partial \sigma} = 0. \tag{54}$$

The coefficient β in equation (39a) is positive. Consequently Z is increasing. The two roots β_1 and β_2 are between 1 and 0. At β_1, Z will be increasing, since $\partial \mathrm{Z}/\partial \beta_1 > 0$, which also shows that $\partial \mathrm{Z}/\partial \sigma = \sigma\beta\,(\beta - 1) > 0$. Equation (54) is valid only if $\partial \beta_1/\partial \sigma < 0$. This implies that the critical level of profit increases with the increase of σ.

The value of the IPO option increases with the increase of the volatility of the profit flows. Moreover, with important fluctuations of this value, the motivation to delay the offer become important. The effect of σ on the critical level of profit is independent of the aversion of the entrepreneur and the decomposition of σ into systematic and specific risk.

2.4 Empirical Simulations

In order to study the interactions between the various parameters influencing the critical level of profit π^* and thus the IPO optimal timing, empirical simulations will be presented. The interactions between the various parameters of the model are based on a sample of companies listed in the French Stock market between 2005 and 2012. These companies belong to four industries. Table 1 shows some descriptive statistics related to these companies, as well as some principal parameters.

Table 1: Descriptive Statistics

		Energy							Finance, Insurance and Real Estate						
		π^*	μ	σ	r^e	x	C	N	π^*	μ	σ	r^e	x	C	N
Mean		85.95	3.6	7	11.6	31.6	8.25	36	157.14	4.1	5.9	11.6	29.9	7.2	20
Min		0.03	3.1	4	7.5	3.4	0		0.63	3.5	2.6	2.7	6.9	0.04	
Perctile	**25^{th}**	1.03	3.2	5.7	8.9	17.4	0.11		2.39	3.7	4.8	9.1	19.9	0.12	
	50^{th}	2.37	3.4	6.3	10.6	25.4	0.40		5.33	4.1	5.9	11.8	26.6	0.44	
	75^{th}	21	4	8	12.3	37.1	2.75		23.8	4.3	6.9	13.4	34	3.22	
Max		762.24	4.7	14.4	21.2	92.8	59.15		1524.48	5.3	8	21.2	77.9	59.9	
Std dev.		197.71	0.4	2.3	3.8	24.6	17.04		392.4	0.4	1.3	3.7	16.4	16.5	
		Manufacturing							**Technology**						
		π^*	μ	σ	r^e	x	C	N	π^*	μ	σ	r^e	x	C	N
Mean		16.01	4.2	5.1	11.1	25.6	5.2	38	192.58	3.8	6.4	13.4	29.5	3.3	18
Min		0.01	3.6	2.2	2.7	3.4	0		0.27	3.2	3.4	7.4	2.6	0.003	
Perctile	**25^{th}**	0.5	3.9	3.5	8	13.6	0.09		23.9	3.5	5	9.3	11.5	0.47	
	50^{th}	1.2	4.1	4.5	10	22.6	0.2		91.01	3.8	5.8	12	24.7	1.44	
	75^{th}	2.49	4.3	66	14	32.5	0.55		142.29	4.1	7.2	18.2	47.4	2.86	
Max		135.4	5.3	10.1	17	83.5	50		1211.71	4.3	14.4	198	92.8	26.1	
Std dev.		38.86	0.4	1.9	3.6	18.5	13.3		313.97	0.3	2.4	0.04	23.3	6.1	

Variables Definition: π^*: Critical profit level in Million Euro (IPO Prospectus), μ: growth rate of profit (IPO Prospectus), σ: the annual sector volatility (datastream), r^d: the entrepreneur discount rate (IPO Prospectus), **x**: proportion of shares issued (IPO Prospectus), **C**: the IPO costs (IPO Prospectus), **N**: number of observations.

The entrepreneur sells a proportion x of its shares during the IPO. According to Chen and Ritter (2000), more than 90% of offers raising between 20 and 80 million dollars are associated with about $\gamma = 7\%$ of underpricing. The model shows that the critical level of profit is positively related to the proportion of the capital issued and negatively to entrepreneur discount rate. Consequently, the analysis of the impact the volatility on the critical level of profit will depend on three ranges of the discount rate r^e for the four sectors of our sample. The companies of the sample employed a discount rate lower than 9%, between 9% and 12% and higher than 12%. The change of these values will have an impact on the level of profit noted by the myopic entrepreneur π^n and consequently on the critical level of profit π^*. However, this will not affect the relative importance of the value of the optimal timing option.

The interpretation of the results of our simulations must be made with caution. Indeed, the change of the values of some parameters implies the change of the values of other parameters. For example, an increase in the volatility, measured by the annual standard deviation of the market index σ, implies the increase in the entrepreneur and market discount rates. In an equivalent way, since σ tends to move towards 0, the discount rates converge. Consequently, the entrepreneur will be less motivated to go public, independently of the value of the optimal timing option.

The impact of volatility on the critical level of profit is illustrated in figures 1, 2, 3 and 4. The value of π^* is measured in million euros. The increase in the volatility of the index leads to the rise of the critical level of profit π^*. This result is valid for the three ranges of the entrepreneurs' discount rates. However, this shows also that the rise of r^e involves the fall of the critical level of profit for any level of volatility for all the sectors.

Moreover, since r^e tends to move towards r^m, the advantage related to the price of risk tends to disappear. The effect of this fall of the advantages of the market valuations become importance when the difference between r^e and r^m is close to zero, since the critical level of profit π^* increases exponentially. However, if r^e and r^m depend on the volatility of the index, the relation between r^e and r^m and the critical level of profit will likely become negative, contrary to the positive relation presented by our results. Thus, the time of waiting before the IPO of the companies belonging to slightly volatile sectors lengthens as regard the difference between r^e and r^m. The opposite is valid for the most volatile sectors.

Figure 1: Relation between π^* and σ
(Energy)

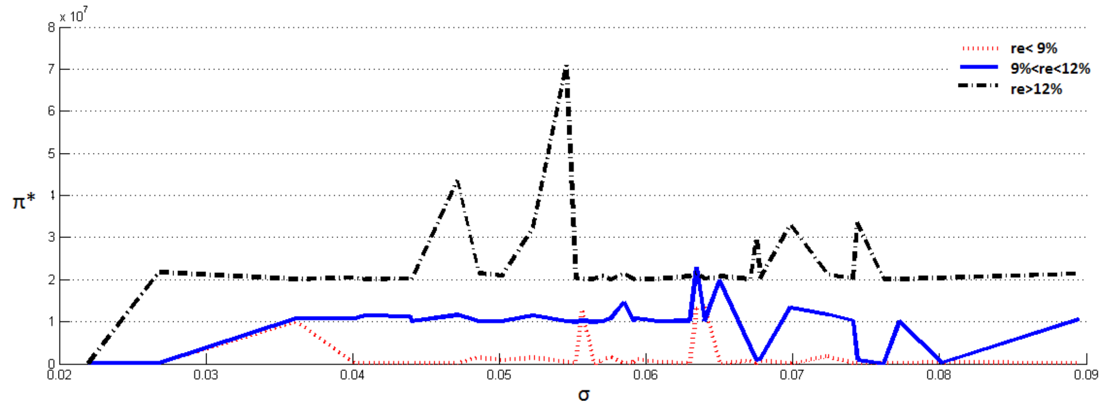


Figure 2: Relation between π^* and σ
(Finance, Insurance and Real Estate)

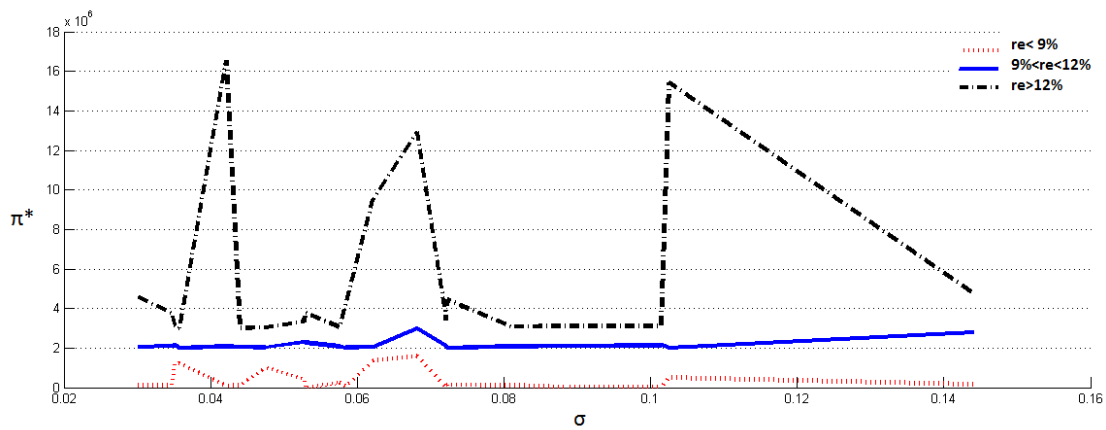


Figure 3: Relation between π^* and σ
(Manufacturing)

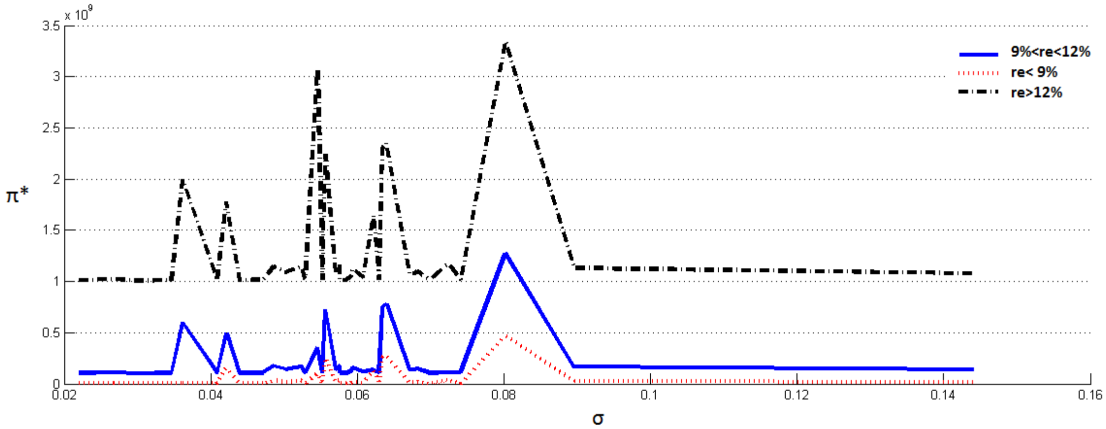
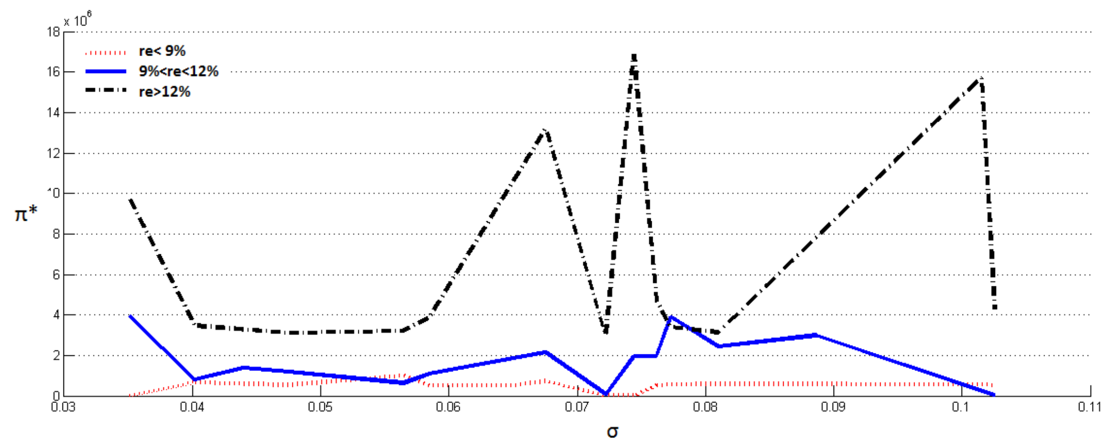


Figure 4: Relation between π^* and σ
(Technology)



Our model goes beyond a myopic entrepreneur. It highlights the alternative that the entrepreneur know well in advance the value of the IPO option as well as the potential value of waiting in order to choose an optimal timing. This result is illustrated in figures 5, 6, 7 and 8. It shows the relation between R^s and the volatility of the 4 indexes of our sample. R^s is the factor with which the critical level profit of a myopic entrepreneur π^n must be multiplied to reach the critical level of profit and then go public. According to our model, R^s is a decreasing function of the level of the entrepreneur discount rate before the IPO and r^e an increasing function of the volatility of the index. The future profits of the candidate company lose thus of their value. Indeed, for a level of volatility of 20%, the level of the critical profit must be eight times higher than the level of profit of a naive entrepreneur π^n.

Without taking into account of the option of optimal timing, the forecasts of the IPO timing will be completely different. Let note that R^s does not depend on r^m. Thus, the decision to go public or to wait depends only on the volatility of the market and of the difference between the discount rates, the cost fixes C and the present value of the future profits.

Figure 5: Relation between R^s and σ
(Energy)

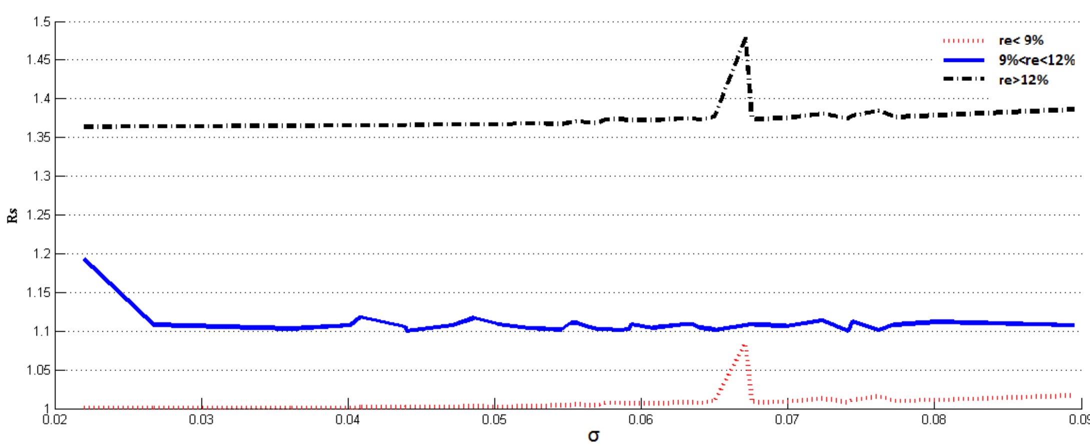


Figure 6: Relation between R^s and σ
(Finance insurance and Real Estate)

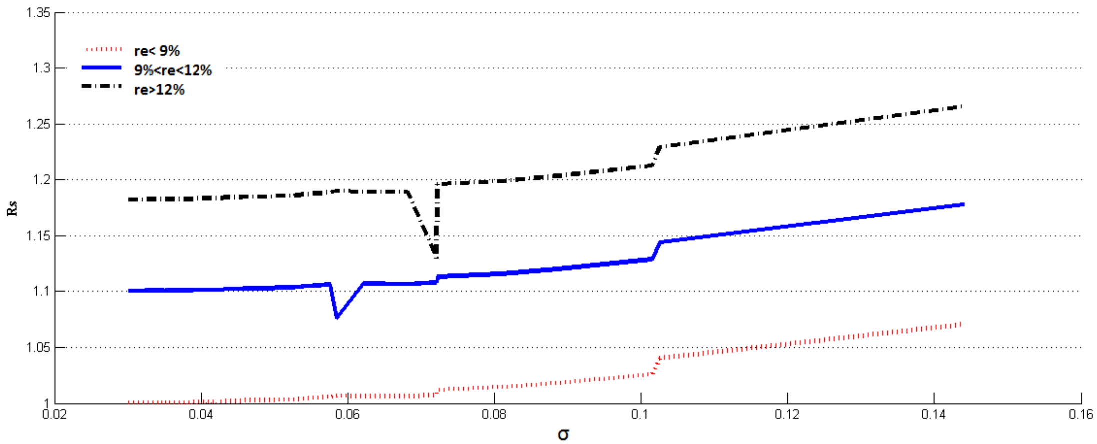


Figure 7: Relation between R^s and σ
(Manufacturing)

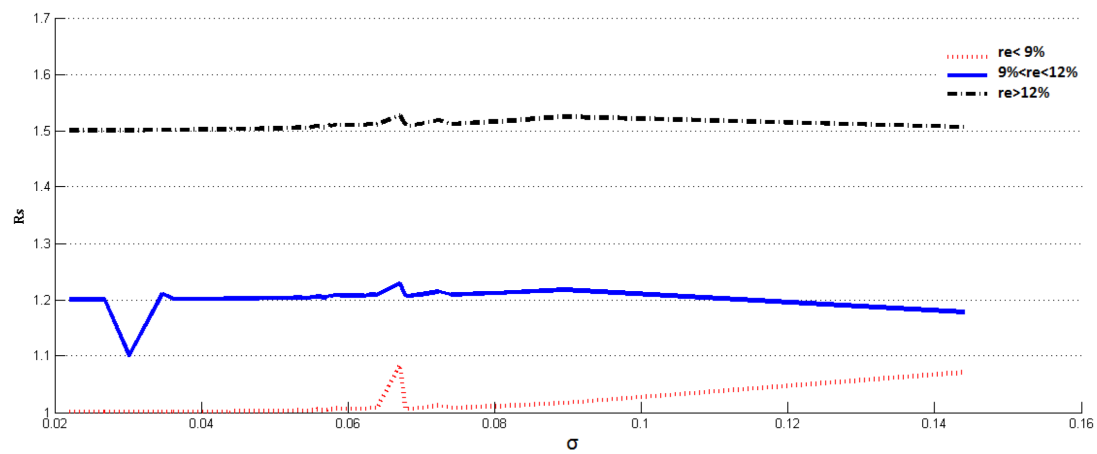
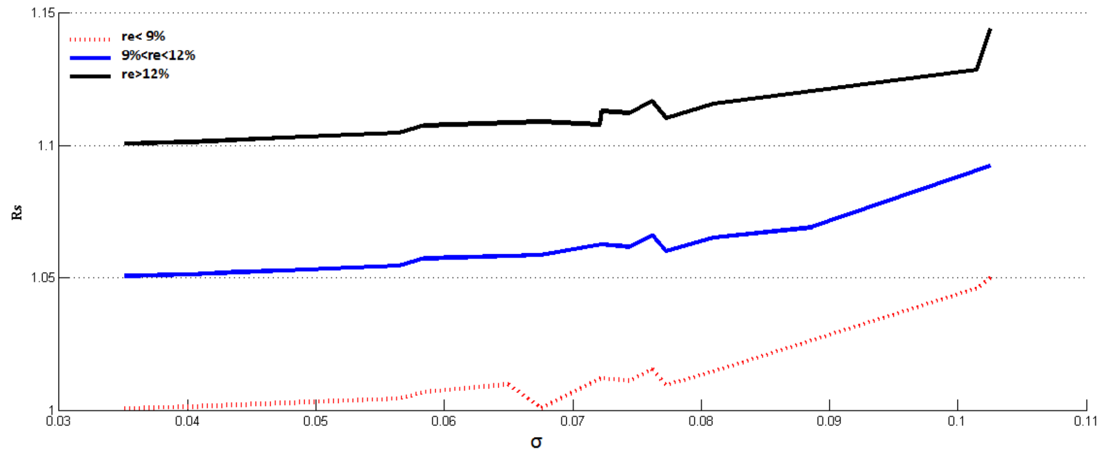


Figure 8: Relation between R^s and σ
(Technology)



In addition, figures 9, 10, 11 and 12 show the impact of the growth rate of the profit on the critical level of profit. Let us note that all the parameters of our model depend on the sector of the candidate companies and on the three ranges of the discount rates.

Figure 9: Relation between π^* and μ
(Energy)

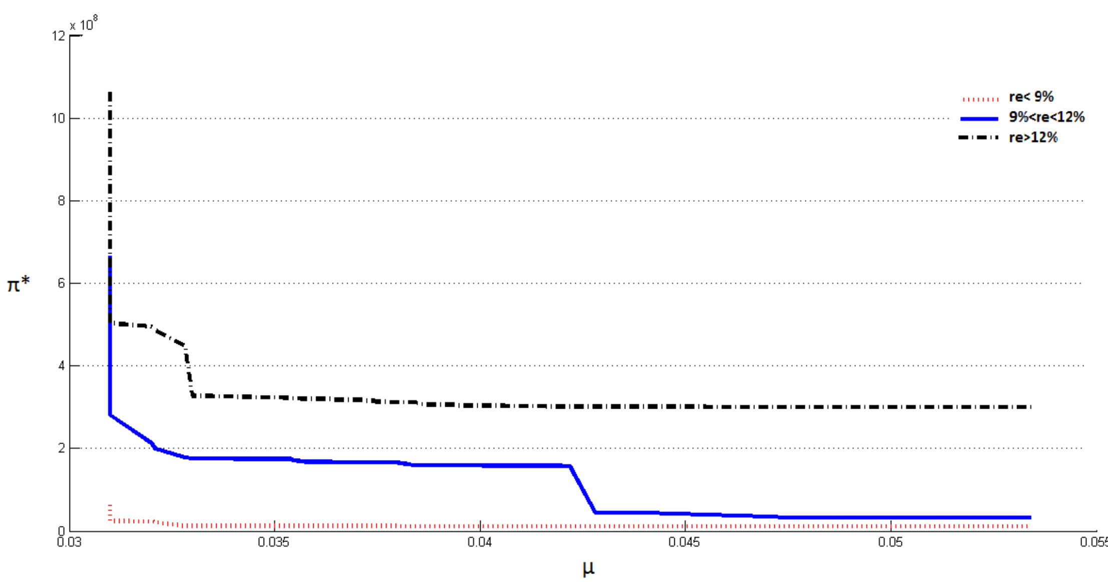


Figure 10: Relation between π^* and μ
(Finance, Insurance and Real Estate)

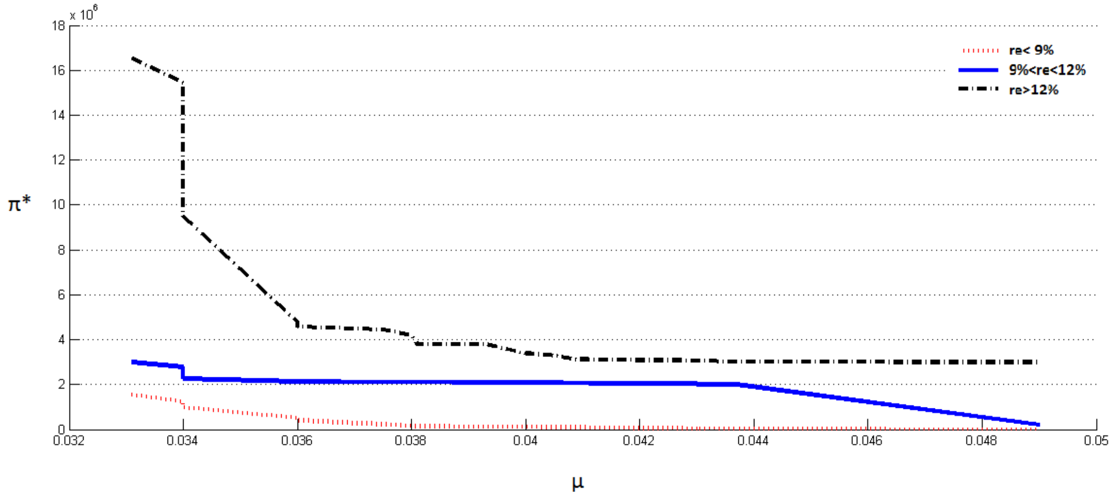


Figure 11: Relation between π^* and μ
(Manufacturing)

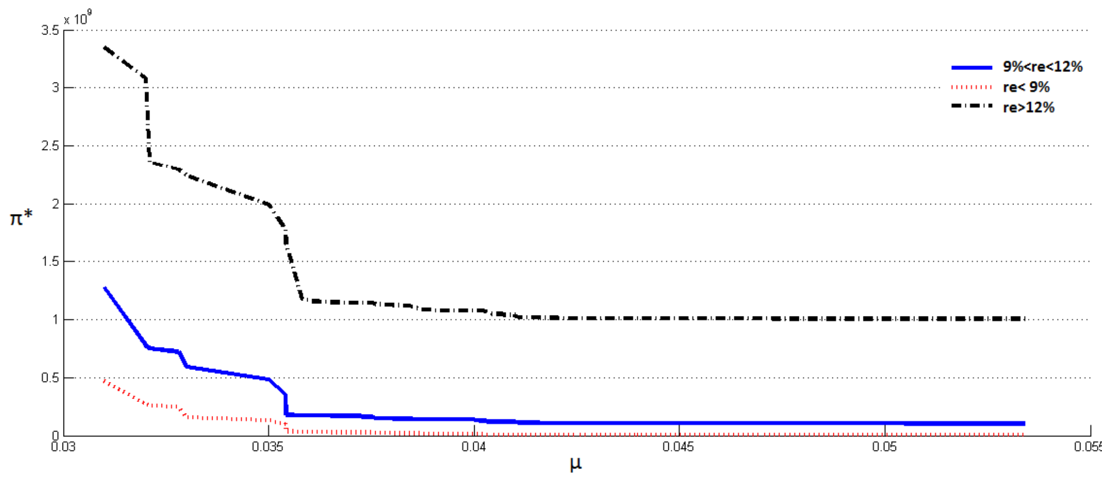
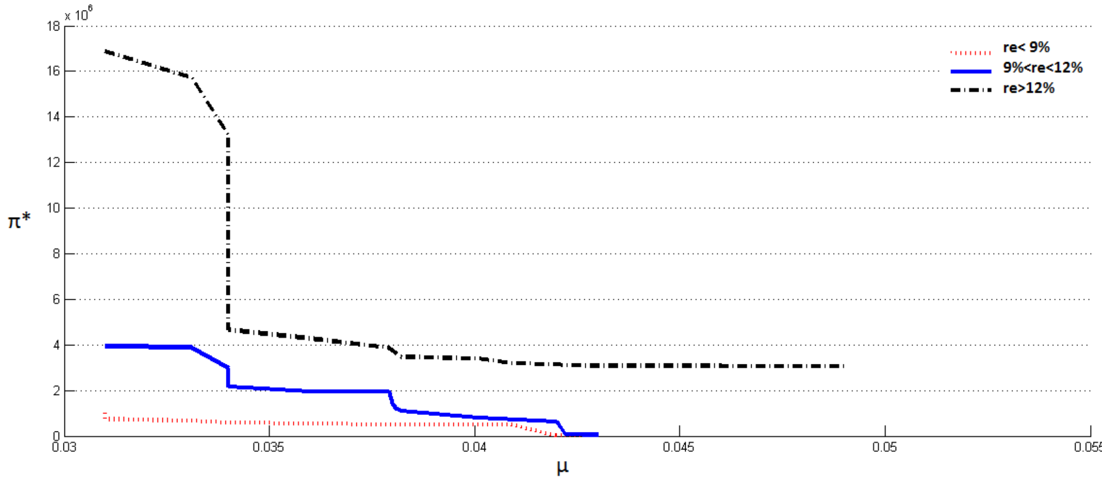


Figure 12: Relation between π^* and μ
(Technology)



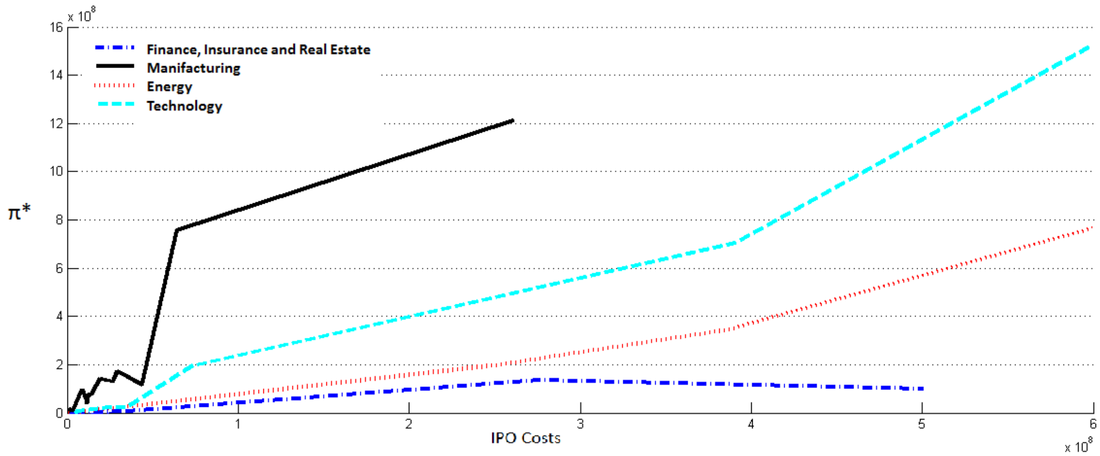
The increase in the growth rate of the profit of the candidate company leads to the reduction in their levels of critical profits π^*. Moreover, the more growth rate of the profit tends towards the market discount rate, the advantage related to the price of risk increases, involving then faster IPOs.

For this reason, two situations can be considered as equivalent. The first is when the entrepreneur discount rate increases and the growth rate of the profit remains constant. The second, for a fixed entrepreneur discount rate, the growth rate of the profit increases. These two situations involve the increase in the R^s factor. Thus, going public corresponds to a higher critical level of profit and consequently a longer waiting time.

Although it is not shown in all the figures, the effect of the increase of the market discount rate, all things being equal, is equivalent to the decrease of the growth rate μ or of the entrepreneur discount rate. The price of the risk decreases thus with the increase of r^m and the company waits longer before the IPO. Consequently, the increase in the market value dominates the increase in the value of the optimal timing option.

Lastly, let's note that according to figure 13, the increase in the costs of the IPO can be regarded as a dissuasion to go public. Indeed the level of the critical profit increases when the costs of the IPO increase. This is valid for the relation between the level of the optimal profit and volatility and for the relation between the level of the optimal profit and the growth rate. These results were verified for all the sectors of our sample.

Figure 13: Relation between π^* and C



Conclusion

This article has thus allowed to model the impact of the flexibility of the IPO optimal timing and the value of the candidate company. When the uncertainty of the profits increases, the value of the option of the optimal timing option becomes increasingly high. An IPO corresponds to the exercise of an expensive option. Thus, the companies have to wait for an increase of market prices before going public in order to cover the costs of the exercise of this option.

In addition, the assumption according to which information is exogenous can be reasonable when we are interested in only one company. However, this assumption becomes restrictive when we analyze a sector. Going public is an informative event for investors, not only regarding the candidate company, but also the entire sector. Moreover, a successful IPO can induce a discrete increase in the values of the remaining private companies (Rajan and Servaes, 1997). An endogenous information can thus allow the joint determination of the increase of market prices and the clustering of IPOs. It is thus possible to show that underpricing is endogenous and this is the result of positive information produced by previous IPOs.

In a general way, information asymmetry has an important role when determining the IPO optimal timing. Indeed, IPOs are realized close to the peaks of market index because the insiders of the candidate company know that the companies of the sector are over-valuated. This is certainly the most important condition that suggests that capital markets are not completely efficient. However, information asymmetry doesnt explain the reasons of the clustering near to market peaks but an over-valuated company have to go public at the peaks of the market index.

The model puts forward that the entrepreneur waits before the IPO, even when the proceeds of the offer are higher than the value of the private company. This difference was supposed considering the difference of the discount rates used by the entrepreneur and the market. However the model shows that a high market value is not sufficient to justify an IPO. By supposing stochastic market prices, we can guarantee that the increase of prices leads to the IPO. Thus, a complete explanation of the motivation of a going public must take into account the IPO optimal timing option and information asymmetry.

The model suggest that the IPO clustering close to the peaks of the market is the consequence of the exercise of the optimal timing option. Although this framework presents a rational explanation to the increase of market prices before an IPO, we cannot conclude as regards their fall after the IPO. The underpricing was analyzed by Ritter (1991) which shows that the shares issued during an IPO present a performance lower than a benchmark of companies from the same sector for a period of three to five years. Similar results were obtained by other studies.[10]

[10] See Peavy (1990), Loughran *et al.* (1994), Loughran and Ritter (1995), Levis (1995), Rajan and Servaes (1997), Baker and Wurgler (2000), Hansen (2001) and Ritter (2003).

References

Allen, F. and G. R. Faulhaber, 1989, "Signaling by Underpricing in the IPO Market," Journal of Financial Economics, 23, 303–323.

Baker, M. and J. Wurgler, 2000, "The Equity Share in New Issues and Aggregate Stock Returns," Journal of Finance, 55, 2219–2257.

Benninga, S., M. Helmantel and O. Sarig, 2005, "The Timing of Initial Public Offerings," Journal of Financial Economics, 75, 115–132.

Benveniste, L. M., W. Y. Busaba and W. J. Wilhelm, 2002, "Information Externalities and the Role of Underwriters in Primary Equity Markets," Journal of Financial Intermediation, 11, 61–86.

Blackwell, D. W. and R. M. Pavlik, 1999, "Marketability and Share Value," Emory University Working Paper.

Brau, J. C. and S. E. Fawcett, 2006, "Initial Public Offerings: An Analysis of Theory and Practice," Journal of Finance, 61, 399–436.

Burgstaller, J., 2009, "When and Why do Austrian Companies Issue Shares?" Empirica, 36, 229–244.

Chemmanur, T. J. and P. Fulghieri, 1999, "A Theory of the Going-Public Decision," Review of Financial Studies, 12, 249–279.

Chen, H.-C. and J. R. Ritter, 2000, "The Seven Percent Solution," Journal of Finance, 55, 1105–1131.

Degeorge, F. and F. Derrien, 2001, "Les déterminants de la performance à long terme des introductions en bourse : le cas français," Banque et Marchés, 55, 8–18.

Dixit, A. K. and R. S. Pindyck, 1994, Investment under Uncertainty, Princeton, NJ, Princeton University Press.

Eckbo, B. E. and Ø. Norli, 2005, "Liquidity Risk, Leverage and Long-run IPO Returns," Journal of Corporate Finance, 11, 1–35.

Gajewski, J.-F. and C. Gresse, 2006, "A Survey of the European IPO Market," European Capital Markets Institute Paper #2.

Hansen, R. S., 2001, "Do Investment Banks compete in IPOs?: The Advent of the '7% Plus Contract'," Journal of Financial Economics, 59, 313–346.

Hart, O. and J. Moore, 1995, "Debt and Seniority: An Analysis of the Role of Hard Claims in Constraining Management," American Economic Review, 85, 567–585.

Ibbotson, R. G. and J. F. Jaffe, 1975, "'Hot Issue' Markets," Journal of Finance, 30, 1027–1042.

Jacquillat, B., 1994, L'introduction en bourse, Paris, Presses Universitaires de France.

Jog, V. M. and A. L. Riding, 1987, "Underpricing in Canadian IPOs," Financial Analysts Journal, 43, 48–55.

Kaplan, S. N., 1991, "The Staying Power of Leveraged Buyouts," Journal of Financial Economics, 29, 287–313.

Kim, W. and M. S. Weisbach, 2005, "Do Firms Go Public to Raise Capital?" National Bureau of Economic Research Working Paper Series #11197.

Latham, S. and M. R. Braun, 2010, "To IPO or Not To IPO: Risks, Uncertainty and the Decision to Go Public: To IPO or Not To IPO," British Journal of Management, 21, 666–683.

Lee, I., S. Lochhead, J. Ritter and Q. Zhao, 1996, "The Costs of Raising Capital," Journal of Financial Research, 19, 59–74.

Leland, H. E. and D. H. Pyle, 1977, "Informational Asymmetries, Financial Structure, and Financial Intermediation," Journal of Finance, 32, 371–387.

Lerner, J., 1994, "Venture Capitalists and the Decision to Go Public," Journal of Financial Economics, 35, 293–316.

Levis, M., 1995, "Seasoned Equity Offerings and the Short- and Long-run Performance of Initial Public Offerings in the UK," European Financial Management, 1, 125–146.

Ljungqvist, A. P. and W. J. Wilhelm, 2002, "IPO Allocations: Discriminatory or Discretionary?" Journal of Financial Economics, 65, 167–201.

Loughran, T. and J. R. Ritter, 1995, "The New Issues Puzzle," Journal of Finance, 50, 23–51.

Loughran, T. and J. R. Ritter, 2002, "Why Don't Issuers Get Upset about Leaving Money on the Table in IPOs?" Review of Financial Studies, 15, 413–443.

Loughran, T., J. R. Ritter and K. Rydqvist, 1994, "Initial Public Offerings: International Insights," Pacific-Basin Finance Journal, 2, 165–199.

MacDonald, J. G. and B. Jacquillat, 1974, "L'efficacité de la procédure française d'introduction en bourse," Banque, 325, 31–39.

Mauer, D. C. and L. W. Senbet, 1992, "The Effect of the Secondary Market on the Pricing of Initial Public Offerings: Theory and Evidence," Journal of Financial and Quantitative Analysis, 27, 55–79.

Mello, A. S. and J. E. Parsons, 1998, "Going Public and the Ownership Structure of the Firm," Journal of Financial Economics, 49, 79–109.

Merton, R. C., 1987, "A Simple Model of Capital Market Equilibrium with Incomplete Information," Journal of Finance, 42, 483–510.

Mikkelson, W. H., M. M. Partch and K. Shah, 1997, "Ownership and Operating Performance of Companies that Go Public," Journal of Financial Economics, 44, 281–307.

Pagano, M., F. Panetta and L. Zingales, 1998, "Why Do Companies Go Public? An Empirical Analysis," Journal of Finance, 53, 27–64.

Pagano, M. and A. Röell, 1998, "The Choice of Stock Ownership Structure: Agency Costs, Monitoring, and the Decision to Go Public," Quarterly Journal of Economics, 113, 187–225.

Pástor, Ľ. and P. Veronesi, 2005, "Rational IPO Waves," Journal of Finance, 60, 1713–1757.

Pavlik, R. M. and W. H. Dare, 2002, "Issuer-oriented Underpricing, Share Marketability, and Share Value," International Journal of Business, 7, 17–28.

Peavy, J. W., 1990, "Returns on Initial Public Offerings of Closed-end Funds," Review of Financial Studies, 3, 695–708.

Pratt, S. P., 2008, Valuing A Business: The Analysis and Appraisal of Closely Held Companies, New York, NY, McGraw Hill.

Rajan, R. G., 1992, "Insiders and Outsiders: The Choice between Informed and Arm's-length Debt," Journal of Finance, 47, 1367–1400.

Rajan, R. G. and H. Servaes, 1997, "Analyst Following of Initial Public Offerings," Journal of Finance, 52, 507–529.

Ritter, J. R., 1987, "The Costs of Going Public," Journal of Financial Economics, 19, 269–281.

Ritter, J. R., 1991, "The Long-Run Performance of Initial Public Offerings," Journal of Finance, 46, 3–27.

Ritter, J. R., 2003, "Differences between European and American IPO Markets," European Financial Management, 9, 421–434.

Ritter, J. R. and I. Welch, 2002, "A Review of IPO Activity, Pricing, and Allocations," Journal of Finance, 57, 1795–1828.

Sahlman, W. A., 1990, "The Structure and Governance of Venture-Capital Organizations," Journal of Financial Economics, 27, 473–521.

Subrahmanyam, A. and S. Titman, 1999, "The Going-Public Decision and the Development of Financial Markets," Journal of Finance, 54, 1045–1082.

Zingales, L., 1995, "Insider Ownership and the Decision to Go Public," Review of Economic Studies, 62, 425–448.