Parsimonious Economic Modelling of an Optimal Sparse Index of Systemic Risk Measures[1]

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ARTICLE INFO	ABSTRACT
JEL classification: G12 G62 H63 *Keywords:* Sparse PCA, Systemic Risk Measures, Financial Crisis.	After the last major financial crisis of 2017-2018, a number of systemic risk measures have been proposed in the financial literature as attempts for quantifying the magnitude of the financial system distress. In this article, we suggest the construction of an overall meta-index for the measurement of systemic risk based on a Sparse Principal Component Analysis of main systemic risk measures, which ultimately aims to provide an index with a more stable dynamic, which is explicitly linked to severe economic recessions.

1. Introduction

The last severe financial crisis of 2007-2008 was characterized both by the speed of financial contagion and by its strong negative impact on the real sector - with a consequent contraction of economic activity in many developed countries. The subsequent European debt crisis have compelled a number of Eurozone members to increase their public spending in order

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to support their respective banking sectors. Under this severe budgetary pressure, some countries encountered difficulties to raise the funds needed to finance their increasing debt, and almost brought the Eurozone on the brink of collapse. In this context, identifying the most systemically important countries has become crucial (Cf. Popescu and Turcu, 2017). If the period preceding the financial crisis was by far characterized by a lack of suitable risk indicators, both practitioners and academics have tried to propose measures able to capture the risk accumulation since the outbreak of the crisis, whilst the first focus was mainly put on systemic risk measures. One way in which the present systemic risk measures can be classified is based on data used to compute them. We can distinguish between, on the one hand, market based measures and, on the other hand, measures which involve confidential information related to the balance sheet and to the financial position of each institution. While the latter are only available to regulators, the former have the advantage of being accessible to all interested parties: academics, practitioners and regulators. However, one of the major issues in financial economics as a result of these turbulent events was, first, to try to agree on one (or more) definition(s) of systemic risk, multifaceted by nature, when emphasizing one or another essential characteristic of financial institutions. Indeed, the intention was to highlight the different aspects of this risk: size of the financial institution in shock, leverage risk and extreme market liquidity shortage phenomenon of interconnections between actors and contagion shock, have been identified among the key elements of a systemic crisis. Once these aspects were identified, the objective was to build relevant analysis tools for measuring the systemic risk. Many authors have proposed measures reflecting both the general state of the system to distinguish the main institutions contributing to the overall risk. The academic literature linked to systemic risk measurement is still growing with recently published measures (*e.g.* Adrian and Brunnermeier, 2016; Pourkhanali et al., 2016; Brownlees and Engle, 2017), and is also accompanied the new imperative by banking regulatory authorities, proposing a number of systemic risk or metric measurements to be followed (Cf. Benoit et al., 2013). Indeed, there are two types of measures: the individual measures that assess systemic risk institutions in isolation and those that are designed to measure the overall systemic risk. In the first category, we can find for instance the Conditional Value-at-Risk (CoVaR) of Adrian and Brunnermeier (2016), the Marginal Expected Shortfall (MES) of Acharya et al. (2013) and Brownlees and Engle (2017), and the SRISK by Acharya et al. (2012) and Brownlees and Engle (2017). In the second group of measures, we find, among the main, the Spillover Index (SI) of Diebold and Yilmaz (2009) and the Dynamic Causality Index (DCI) of Billio et al. (2012).

However, recent works showed that the definition of a “good” measure of systemic risk still remains unresolved, 1) because of the observed empirical redundancies in the various measures of systemic risk, 2) because of the model risk associated with their estimates and 3) due to the absence of an objective criterion to tell us about the relevance of the different approaches. Thus, recent articles have focused on the implied model risk in the implementation of these different metrics. Daníelsson et al. (2016) and Benoit et al. (2017) show that a large majority of individual measures of systemic risk strongly depends upon extreme percentiles of returns, and therefore inheriting the risk of such uncertain quantities. Actually, model risk seems to be largely underestimated in practice (Boucher et al., 2014 and 2016), and it is heterogeneous in the different steps, leading to significant discrepancies in the rankings of systemic institutions (Benoit et al., 2013; Nucera et al., 2016; Kouontchou et al., 2017). Consequently, measures of overall systemic risk constructed as a weighted sum of individual measures are also subject to model risk (see Moreno and Peña, 2013).

A solution that recently appeared in the literature on systemic risk to mitigate the model risk is to construct aggregate indexes from different existing metrics (e.g. Illing and Liu, 2006; Holló et al., 2012; Louzis and Vouldis, 2012; Giglio et al., 2016; Kouontchou et al., 2017). The main objective is to obtain a risk index which diversifies the model risk. As part of the quantification of the overall systemic risk, this approach is, in particular, adopted by Giglio et al. (2016) who identify an aggregate index at a given date as the common signal extracted from time-series of various metrics of systemic risk, recovered through a Principal Component Analysis (PCA). In their empirical investigations, it appears that this index predicts periods of sharp slowdown in economic activity - which is ultimately the economic criterion that should be the most important. Regarding the individual measures, Nucera et al. (2016) adopt a similar technical approach and they infer a rating issued from other noisy and divergent rankings of competing measures. Beyond the diversification of the model risk, it should be noted that the aggregation also allows us to synthesize the different dimensions of systemic risk (size, leverage, interconnections, liquidity etc.).

In the first step, we retain as a construction tool for the aggregate index: the “Sparse” Principal Component Analysis (SPCA). This method of dimension reduction, as opposed to the standard PCA used in Giglio et al. (2016) in the building of systemic risk indexes, allows us to select a predefined number of active components in the index. In this case, it has the advantage to retain, for the construction of the aggregated risk index, the measures that best explain some output targeted data observations.

The second step of our methodology when building an optimal Index is devoted to the endogenization of the smoothing parameter that governs the scarcity of the SPCA. In our case, the optimal value of this parameter is obtained by retaining the aggregate index that Granger causes extreme variations in economic activity. The inference is here performed using the nonparametric test of causality in extreme risks of Hong et al. (2009). This approach has the advantage to test for a large number of time windows in the causal relation, with higher discounting the longest horizons.

The rest of this article is organized as follows. In the next part, we present the construction method of a systemic risk index with, in the following part, an illustration on the US market. A last part is dedicated to some robustness tests, whist the ultimate section concludes.

2. Using a Sparse Principal Component on Systemic Risk Measures

To integrate systemic risk as another factor, complementary to the systematic and specific risk, it is necessary to use a precise measurement of risk. In recent years, a strong literature has been developed on identification of Systemically Important Financial Institutions by quantitative measures to characterize the conditional link between different financial institutions and the market as a whole. However, given the many dimensions of systemic risk, these individual measures hardly detect systematically the potentially systemic institutions.

The use of factor analysis as information aggregation tool from a set of systemic risk measures is a new approach. Moreno and Peña (2013) use a Principal Component Analysis (PCA) on a set of companies for building a systemic risk index. Giglio et al. (2016) use a principal component analysis to build a systemic risk index and test its predictive power of future shocks on macroeconomic variables using quantile regression. Nucera et al. (2016) run principal component analysis on a set of six systemic risk measures. However, their study differs from Giglio et al. (2016) as they apply a PCA on the ranking of 113 companies in the financial sector through a series of systemic risk measures and not on a set of companies over a period of time as in Giglio et al. (2016) in their study from a set of 19 measures of systemic risk. We hereafter summarize, complement and extend the work by Giglio et al. (2016) and Nucera et al. (2016), mainly considering the databases first used in Brownlees and Engle (2017) and, secondly, by Giglio et al. (2016).

2.1. About Systemic Risk Measures

The financial literature has proposed numerous quantitative measures that can be used to identify potentially systemic institutions. We can group them into several categories.
First, the individual systemic risk measures are defined from econometric models of specific risk to the institution. This is the Conditional Value-at-Risk (CoVaR) and Delta Conditional Value-at-Risk (ΔCoVaR) by Adrian and Brunnermeier (2016), and the Marginal Expected Shortfall (MES) of Acharya et al. (2013) and Brownlees and Engle (2017), and SRISK of Acharya et al. (2012) and Brownlees and Engle (2017), and Amihud Illiquidity Measure proposed by Amihud (2002).

Secondly, other measures focus specifically on an important aspect of systemic risk, i.e. the level of interconnection of financial institutions or the financial system concentration. In this category, we select the Spillover Index (SI) by Diebold and Yilmaz (2009), the Dynamic Causality Index (DCI) of Billio et al. (2012), the measurement of Turbulence by Kritzman and Li (2010), the Absorption Ratio of Kritzman et al. (2011), and the concentration Herfindahl-Hirschman Index. Thirdly, some macro-financial variables are generally used to complement the analysis, serving as leading indicators of economic activity (see Estrella and Trubin, 2006; Chen et al., 2009). We retain hereafter in our analysis: the Credit Default Yield Spread which measures the difference between the yield on corporate bonds rated BAA and the rated AAA by Moody's, as Chen et al. (2009) show that this variable is an aggregate measure of the risk of robust credit frictions (tax and liquidity) in the bond market; the TED Spread, which measures the difference between the LIBOR three-month rate and sovereign interest rates in three months: an increase of this variable is the sign that lenders expect an increase in credit risk in the interbank lending market; and finally the Term Spread measures the slope of the yield curve, and corresponds to the yield spread between 10-year Treasury bonds and three months money market maturities, since this variable may serve as a leading indicator of the economic activity (e.g. Estrella and Trubin, 2006).
We consider also the volatility (Vol) and the Value-at-Risk (VaR) aggregated across the system to take into account the evolution of its variability.

We illustrate the dynamics of these different systemic risk indicators in the following Figure 1 from daily data financial institutions from the US market over the period from the 09/03/2003 to the 02/26/2016. We see in this Figure a significant increase in all global systemic risk measures over the period 2007-2008 which is the period of the financial crisis. Similarly, although a common trend seems to emerge from the dynamics of the series, there are still some disparities between these measures. These differences may stem from the fact that systemic risk is multidimensional, each of the different metrics is modelling a specific dimension.

This result is confirmed by the analysis of correlations between different risk measures.

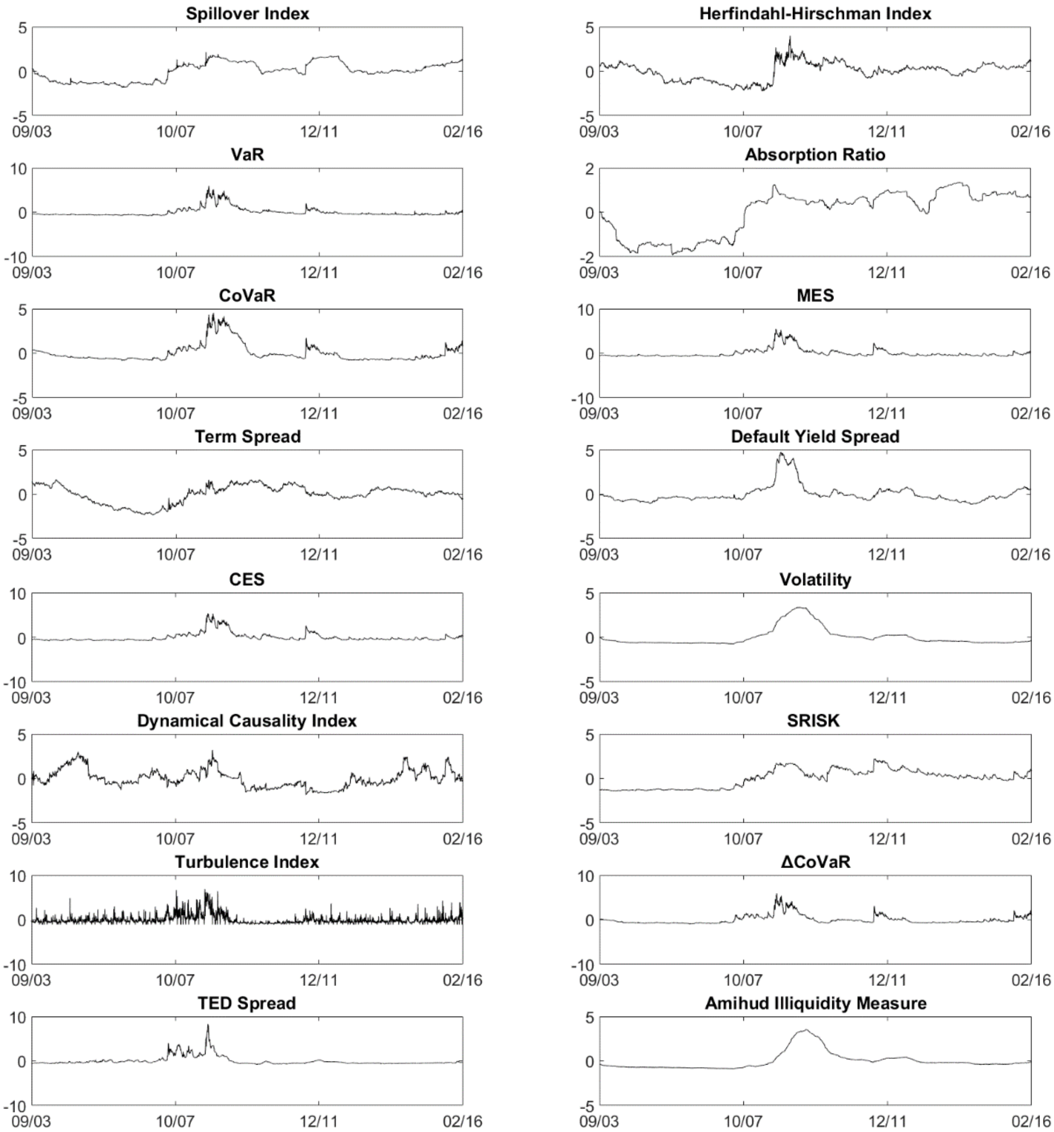

Fig. 1. Dynamics of global systemic risk measures. Note: Bloomberg, daily data from the 09/03/2003 to the 02/26/2016; authors' computation. These measures (without considering the macroeconomic variables) are estimated from rolling windows of one year. Here are presented the z-scores of these measures and are in the following order: M_1: Spillover Index, M_2: Herfindahl-Hirschman Index, M_3: VaR, M_4: Absorption Ratio, M_5: CoVaR, M_6: MES, M_7: Term Spread, M_8: Default Yield Spread, M_9: CES, M_{10}: Volatility, M_{11}: Dynamical Causality Index, M_{12}: SRISK, M_{13}: Turbulence Index, M_{14}: ΔCoVaR, M_{15}: TED Spread and M_{16}: Amihud Illiquidity Measure.

Indeed, from the matrix of the correlations presented in Appendix C, we can note that all correlations are statistically significant at a nominal risk level of 5%, except for the correlations between the Amihud Illiquidity Measure and the TED Spread and between the

Dynamical Causality Index and the Default Yield Spread for the Pearson correlations. The exception of the correlations for the Spearman correlations are between the TED Spread, the aggregate SRISK and the aggregate volatility, between the Turbulence Index and the aggregated SRISK, between the Herfindahl-Hirschman Index and the Dynamical Causality Index and between the aggregated CoVaR and the Dynamical Causality Index.

The strongest correlations (above .90) are those related to global systemic risk measures corresponding to average data instant time-series of individual measures (CoVaR, ΔCoVaR, MES). It is therefore necessary to propose an indicator that integrates all dimensions of risk (aggregated model).

2.2. An Aggregated Index of Systemic Risk Measures

We present in this section the methodology for the construction of the aggregated index of overall systemic risk (see Kouontchou et al., 2017). First, we present the Sparse Principal Component Analysis (SPCA) approach for dimension reduction, and secondly, we present the optimal choice of systemic risk index through the causality test in extreme risks of Hong et al. (2009) for selecting the most parsimonious index.

The PCA is a decomposition of a data set on the basis of orthogonal functions which are determined from the data. These functions, which are linear combinations of the original variables, are supposed to reproduce a large extent of the existing variability in the data, and they correspond to the most important main axes or components. From a statistical point of view, if we consider a matrix M of dimension (T, p) of the initial normalized data (see Benoit et al., 2013 on the importance of this point in the context of detecting Systemic Important Financial Institutions), the first component (main axis) is denoted by a vector of dimension p as the solution of the following program:

$$\begin{gathered}\max_{x \in \mathbb{R}^p} \{x' A x\} \\ s.t. \|x\|_2 = 1,\end{gathered} \tag{1}$$

where $A = T^{-1} M' M$ is the covariance matrix of M of dimension (p, p), M' is its transpose, T the size of the sample and $\| x \|_2$ stand for the 2-norm of vector x.

The first component is obtained by minimizing the empirical variance of the projected data in an identification constraint associated with a specific norm. The projection data on this component makes it possible to obtain a factor noted F of dimension (T, p), with $F = Mx$ whose variance, called eigenvalue, is equal to $\lambda = T^{-1} F' F$, is the criterion in the optimization program (1). In the construction of aggregated systemic risk indices, the index is generally associated with the factor F (Moreno and Peña, 2013; Giglio et al., 2016).

The previous optimization program that provides the first component and the dominant factor has an equivalent representation in terms of linear regression (Zou et al., 2006; Shen and Huang, 2008). Indeed, it is shown that in the linear regression that reads:

$$F = M\beta + U, \tag{2}$$

where the dependent variable F (respectively, the explanatory variables M) is the dominant factor of the PCA (initial data matrix respectively) and U is the error term. The normalized value of the Ordinary Least Square (OLS) estimator of the parameter vector β is equal to the first component, that is:

$$x = \frac{\hat{\beta}}{\|\hat{\beta}\|_2}, \tag{3}$$

with $\|.\|_2$ the 2-norm.

Zou et al. (2006) propose to modify the linear regression represented by Eq. (2) in order to obtain the main sparse component from the expression (3). Indeed, if x^s is this component, it is equal to:

$$x^s = \frac{\hat{\beta}^s}{\|\hat{\beta}^s\|_2}, \tag{4}$$

where $\hat{\beta}^s$ is the solution of the constrained following regression (or penalized) below:

$$\begin{gathered} F = M\beta^s + U \\ s.t. \|\beta^s\|_1 = \sum_{j=1}^{p} |\beta_j^s| \leq \delta. \end{gathered} \tag{5}$$

The parameter $\delta \geq 0$ defines the upper limit of norm 1 of the parameter vector β^s. Regression (5) introduced by Tibshirani (1996) is known as the Least Absolute Shrinkage and Selection Operator (LASSO), and its primary goal is to make a variable selection. The limit behaviour of this regression can be summarized as follows. When δ tends to zero, the number of active elements (different from zero) in $\hat{\beta}^s$, and therefore in the "sparse" component x^s, also approaches zero - the degenerated limit case being when $\delta = 0$, where $\hat{\beta}^s$ and x^s correspond to the zero vector; in the opposite case when δ tends to infinity, the regression (5) is the unrestricted regression (2), and x^s is exactly equal to x, i.e. the main component of a conventional PCA and the number of active elements then takes its maximum value p.

3. Empirical Results

We previously illustrated the dynamics of the different systemic risk measures in the Figure 1 from daily data financial institutions from the US market over the period from the 09/03/2003 to the 02/26/2016.

3.1. Constructing Competing Sparse Indexes of Systemic Risk Measures

This method applied in our systemic risk framework has the advantage of providing a main component that summarizes the variability in systemic risk indicators, using only a few of them. Beyond the parsimony that brings the SPCA, it should be mentioned that the variable selection is made using the usual trade-off between bias and variance. Indeed, under the usual conditions of exogeneity of the error term U in the regression (2), the estimator $\hat{\beta}$ is unbiased. The additional constraint in regression (5) helps to reduce the variance of the estimator by introducing bias.[3]Therefore the main factor from a SPCA has a more stable temporal dynamic. As highlighted above, this property is desirable since the implementation of regulatory policies should not be based on noisy and erratic metrics of systemic risk. Finally, note that the dominant factor of the SPCA is obtained by projecting the data matrix M on the sparse component x^s, with $F^s = Mx^s$.

Table 1 shows the dominant principal component derived from the SPCA methodology for different values of the parameter δ. When $\delta = 1$, which corresponds to the strongest constraint in regression (5), the number of active global systemic risk measures in the dominant component is equal to $k = 1$, and corresponds to the M8 measure, namely the Spillover Index of Diebold and Yilmaz (2009). When δ increases, the constraint becomes lighter and other additional systemic risk measures enter into the dominant component. For illustration, when $\delta = 1.933$, six measures are active in the index, namely the concentration Herfindahl-Hirschman Index, the Absorption Ratio of Kritzman et al. (2011), the Spillover Index of Diebold and Yilmaz (2009), the aggregated MES[4] by Acharya et al. (2010), the aggregated

[3] Here we find a compromise between bias and variance in the so-called regression “RIDGE”. The arbitrage nonetheless addresses the norm 1 and not the norm 2 of the “LASSO” regression.

[4] For some authors, and since its definition relies on the effect on a financial institution of an extreme market movement, the MES should not be taken as a systemic risk measure... We see here that the information content of such a statistic is singular and different from the one in other measures.

Value-at-Risk and the aggregated CoVaR of Adrian and Brunnermeier (2016). For the highest value, all measures are active in the dominant component, and it is the first component of a classic PCA. Note that the latest systemic risk measure to be active in component is the Amihud Illiquidity Measure.

Table 1
Variable decomposition of the sparse principal components

δ	1.00	1.30	1.40	1.71	1.83	1.93	2.01	2.02	2.05	2.12	2.139	2.19	2.24	2.30	2.32	2.38
Id_k	Id_1	Id_2	Id_3	Id_4	Id_5	Id_6	Id_7	Id_8	Id_9	Id_{10}	Id_{11}	Id_{12}	Id_{13}	Id_{14}	Id_{15}	Id_{16}
k	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9	k=10	k=11	k=12	k=13	k=14	k=15	k=16
$\mathbf{M_1}$	1.00	.92	.91	.83	.79	.75	.73	.73	.72	.70	.70	.70	.70	**.69**	.69	.68
$\mathbf{M_2}$	.00	.39	.41	.49	.51	.53	.54	.54	.54	.55	.55	.55	.55	**.56**	.57	.57
$\mathbf{M_3}$	.00	.00	.08	.23	.24	.23	.20	.19	.19	.16	.15	.15	.13	**.10**	.08	.05
$\mathbf{M_4}$	.00	.00	.00	.17	.24	.30	.34	.34	.35	.38	.39	.40	.40	**.40**	.41	.41
$\mathbf{M_5}$	.00	.00	.00	.00	.05	.10	.13	.13	.14	.14	.14	.13	.12	**.07**	.06	.07
$\mathbf{M_6}$	.00	.00	.00	.00	.00	.02	.06	.06	.06	.05	.04	.05	.05	**.04**	.04	.05
$\mathbf{M_7}$	.00	.00	.00	.00	.00	.00	.02	.02	.03	.04	.05	.05	.06	**.06**	.06	.07
$\mathbf{M_8}$	.00	.00	.00	.00	.00	.00	.00	.00	.01	.03	.04	.05	.05	**.07**	.07	.08
$\mathbf{M_9}$	.00	.00	.00	.00	.00	.00	.00	.00	.01	.05	.06	.05	.06	**.06**	.05	.05
$\mathbf{M_{10}}$	.00	.00	.00	.00	.00	.00	.00	.00	.00	.01	.01	.03	.04	**.08**	.10	.06
$\mathbf{M_{11}}$	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.03	.04	**.06**	.06	.07
$\mathbf{M_{12}}$	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.02	.03	**.04**	.05	.06
$\mathbf{M_{13}}$	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.01	**.03**	.03	.04
$\mathbf{M_{14}}$	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	**.05**	.06	.05
$\mathbf{M_{15}}$	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	**.00**	.01	.04
$\mathbf{M_{16}}$	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	**.00**	.00	.05

Note: Bloomberg, daily data from the 09/03/2003 to the 02/26/2016; authors' computation. M_1 to M_{16} represent the 16 systemic risk measures and are in the following order: M1: Spillover Index, M_2: Herfindahl-Hirschman Index, M_3: VaR, M_4: Absorption Ratio, M_5: CoVaR, M_6: MES, M_7: Term Spread, M_8: Default Yield Spread, M_9: CES, M_{10}: Volatility, M_{11}: Dynamical Causality Index, M_{12}: SRISK, M_{13}: Turbulence Index, M_{14}: ΔCoVaR, M_{15}: TED Spread and M_{16}: Amihud Illiquidity Measure.

Figure 2 shows the dynamics of all 16 aggregate indexes of systemic risk obtained through the analysis of the main sparse components for a given value of the truncation parameter δ, varying from $\delta = 1.000$ to $\delta = 2.388$. For the first value, when $\delta = 1.000$, the aggregate index is nothing else than the Spillover Index of Diebold and Yilmaz (2009) and is the most stable. For the last value, where $\delta = 2.388$, the aggregate index corresponds to the aggregation of all systemic risk measures (the 16 measures are included in the analysis). The dynamics of the other indices are between these two limit case indexes. Indeed, the addition of any extra factor in the index increases its variability, which will be between the variability of the two indexes included in the limiting cases as displayed in the following Figure.

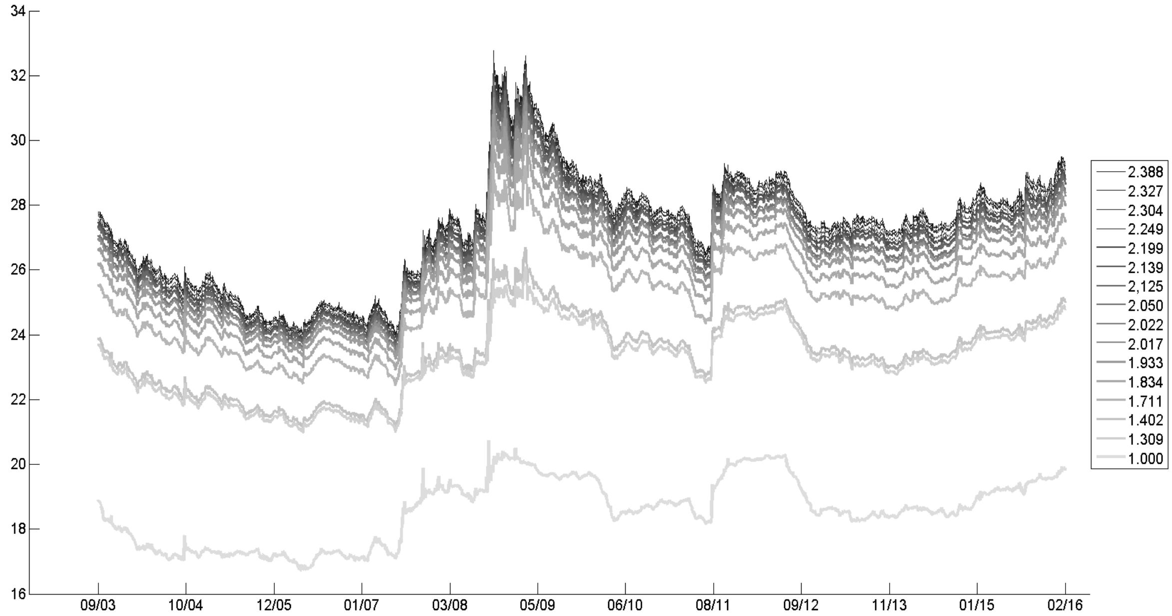

Fig. 2. Dynamics of the various SPCA indexes (as functions of δ). Note: Bloomberg, daily data from the 09/03/2003 to the 02/26/2016; authors' computation. Are presented the dynamics of all the aggregate indexes obtained from SPCA for a specific value of δ.

Figure 3 below, displays the dynamics of the SPCA and PCA cases of the 16 factors (or aggregate indexes) from the sparse components of Table 1. Indeed, although the dynamics of the two aggregate indices are very similar, the temporal variability is not equal. The most stable index obviously corresponds to the case with a variance equal to 1. This index is identical to the Spillover Index of Diebold and Yilmaz (2009). The most volatile aggregate index is obtained for $\delta = 2.388$ and is equal to the dominant factor of a conventional PCA, with an estimated variance of 3.195. Other unrepresented aggregated indexes show variances between these two values. We thus find, with this analysis, the primary objective of the SPCA, namely the temporal stabilization of factors, is to define our aggregate index. However, as already mentioned, this stabilization is achieved *via* a bias-variance arbitrage, and thus induces a decrease in the quality of the representation (explained variance).

Fig. 3. Dynamics of the SPCA and PCA indexes. Note: Bloomberg, daily data from the 09/03/2003 to the 02/26/2016; authors' computation. Are plotted the two limit cases of the set of indexes. The optimal choice is in bold and the index obtained by the PCA is in dashed line.

The following section is dedicated to the optimal choice of the aggregate index of systemic risk among the 16 competing indices which are all special combinations of our 16 systemic risk measures.

3.2. Optimal Choice within the Set of the Competing Sparse Principal Components

Our aim in this section is to assess to what extent the aggregate index can be considered a leading indicator of economic activity. This approach is the one used by Giglio et al. (2016) to measure the predictive power of their aggregate index extracted from a classical PCA. Indeed, these authors, *via* a quantile regression, test whether extreme variations in the industrial production is explained by the lagged value of the index of systemic risk - compared to a non-conditional specification excluding the index.

The method we adopt in this section is, however, different, in the sense that we assess to what extent the positive extreme movements of the aggregate index of systemic risk (when systemic risk is high) Granger-cause the negative extreme movements in the industrial production. As highlighted above, this approach is consistent with the intuition that only the extreme movements of the aggregate index can explain systemic events, inducing strong slowdowns in the future economic activity. We use for this purpose the causality test in distributions tails developed by Hong et al. (2009).

For a brief description of the test, let us note$y_{1,t} = \Delta P_t$ the monthly change in industrial production, and $Q_{1,t}(\alpha;\theta_1)$ the quantile at the order α of the distribution of $y_{1,t}$, with θ_1 a vector of parameters associated with the specification of the dynamic of $y_{1,t}$. Here we follow Giglio et al. (2016) by setting α to 20%. For monthly data, note here this is a reasonable choice since it allows to have samples with limited sizes and a significant number of observations in the left tail of the distribution y_t. Let $Hit_{1,t}(\alpha;\theta_1)$ the dummy variable defined as:

$$Hit_{1,t}(\alpha;\theta_1) = \begin{cases} 1\ if\, y_{1,t} \leq\ Q_{1,t}(\alpha;\theta_1) \\ 0\ otherwise \end{cases}. \tag{6}$$

This variable equals 1 when the change in industrial production is extreme and negative, corresponding to a severe contraction of economic activity. In the same manner, let us denote $y_{2,t} = -\Delta F_t^s$ the opposite of the monthly change in the aggregate index of systemic risk[5] obtained *via* the PCA « *sparse* » methodology, and $Hit_{2,t}(\alpha;\theta_2)$ the dummy variable defined as:

$$Hit_{2,t}(\alpha;\theta_2) = \begin{cases} 1\ if\, y_{2,t} \leq\ Q_{2,t}(\alpha;\theta_2) \\ 0\ otherwise \end{cases}. \tag{7}$$

Note that this variable equals 1 when the change in aggregated systemic index is extreme and positive indicating a systemic event. The null hypothesis testing in Hong et al. (2009) is:

$$E\left[Hit_{1,t}(\alpha;\theta_1)|\Omega_{t-1}\right] = E\left[Hit_{1,t}(\alpha;\theta_1)|\Omega_{1,t-1}\right], \tag{8}$$

wherein the sets of information on the date t-1 are defined respectively by:

$$\begin{cases} \Omega_{t-1} = \left\{\left(y_{1,s}, y_{2,s}\right), s \leq t-1\right\} \\ \Omega_{1,t-1} = \left\{y_{1,s}, s \leq t-1\right\}. \end{cases} \tag{9}$$

Under the null hypothesis, the positive extreme movements of the aggregate index of systemic risk have no predictive power on the negative extreme movements in industrial production. The test statistic proposed by the authors depends on a weighted sum of the estimated correlations between $Hit_{1,t}\left(\alpha;\hat{\theta}_1\right)$ and $Hit_{2,t}\left(\alpha;\hat{\theta}_2\right)$ where $\hat{\theta}_1$ and $\hat{\theta}_2$ are consistent estimators of $\theta_1$and θ_2. This weighted sum is defined by:

$$Z = T\sum_{j=1}^{T-1} \kappa^2(j/d)\hat{\rho}(j), \tag{10}$$

[5] Monthly data for each aggregate index are obtained as averages of daily data of Figure 2. In total, we have 130 observations for competitor aggregate indices, and 130 monthly observations for industrial production.

With the function $\kappa(.)$ of the type decreasing kernel[6], d the truncate parameter[7] and $\hat{\rho}(j)$ the cross-correlation of order j between $Hit_{1,t}(\alpha;\hat{\theta}_1)$ and $Hit_{2,t}(\alpha;\hat{\theta}_2)$ equals to:

$$\hat{\rho}(j) = \frac{\hat{\gamma}(j)}{\hat{s}_1\hat{s}_2}, \tag{11}$$

where $\hat{s}_1$ and $\hat{s}_2$ refer to the standard deviation of $Hit_{1,t}(\alpha;\hat{\theta}_1)$ and $Hit_{2,t}(\alpha;\hat{\theta}_2)$ respectively and $\hat{\gamma}(j)$ cross-covariance of order j defined by:

$$\hat{\gamma}(j)\begin{cases} T^{-1}\sum_{t=l+j}^{T-1}\{[Hit_{1,t}(\alpha;\hat{\theta}_1)-\hat{\pi}_1][Hit_{2,t-j}(\alpha;\hat{\theta}_2)-\hat{\pi}_2]\}\ for\ 0\le j\le T-1 \\ T^{-1}\sum_{t=l-j}^{T-1}\{[Hit_{1,t+j}(\alpha;\hat{\theta}_1)-\hat{\pi}_1][Hit_{2,t}(\alpha;\hat{\theta}_2)-\hat{\pi}_2]\}\ for\ 1-T\le j\le 0, \end{cases} \tag{12}$$

with $\hat{\pi}_1$ and $\hat{\pi}_2$ the empirical means of $Hit_{1,t}(\alpha;\hat{\theta}_1)$ and $Hit_{2,t}(\alpha;\hat{\theta}_2)$ respectively. We therefore denote that the particularity of the Z statistic is the fact that all possible lags are considered, with a discount of the most distant lags. Also, in the current context of applying this test, the inclusion of a high number of lags, helps to capture the stronger or weaker inertia in the reaction of the economy to a systemic event. Under the null hypothesis of no causality in extreme movements, Hong et al. (2009) demonstrate that:

$$U = \frac{Z - C_T(d)}{[D_T(d)]^{1/2}}, \tag{13}$$

follows a standard normal distribution, with:

$$C_T(d) = \sum_{j=1}^{T-1}(1 - j/T)\kappa^2(j/d), \tag{14}$$

and:

$$D_T(d) = 2\sum_{j=1}^{T-1}(1 - j/T)(1 - (j+1)/T)\,\kappa^4(j/d). \tag{15}$$

The U-statistic is therefore used for inference. The Monte Carlo simulations carried out by Hong et al. (2009) show that the test has good properties at a finite distance. It is important here to note that the minimum sample size considered by the authors in the simulations is$T\ =$

[6]. We use the kernel function from Daniell that induces optimal properties for causality test. Cf. Hong et al. (2009) for further details.

[7]. When this parameter d increases, the value of the function that plays in the formula (10) as weighting is higher for low values of j lags.

500, and the minimum quantile is 5% (approximately 25 observations in the tails of distributions). We have here with our monthly data of the changes in industrial production and changes in competitors aggregated indices only 129 observations. With a 20% quantile, this leaves us also 25 cases. It is close to the test application conditions, namely the existence of a relatively not too small number of data in the tails of distributions.

The results of causality tests for the different competing indices (denoted Id_1 to Id_{16}) are summarized in Table 2, for two values of truncation parameter d, ranging from $d = 10$ and $d = 25$. The null hypothesis of no causality from positive and extreme monthly variations of each aggregate index of systemic risk to the negative and extreme monthly variations in industrial production, is rejected in all configurations at a nominal 5% threshold. When closely reading this Table, the optimal index derived from the SPCA methodology is the aggregate index 14. Indeed, whatever the value of d, this index appears to be the most parsimonious: it is constructed from only 14 systemic risk measures, and is relatively stable over time, whist it has the highest predictive power (high test statistic) on severe contractions in the economic activity. We note here the analogy between our approach to identify the optimal aggregate index and the traditional model selection *criteria* (AIC, BIC).

Table 2
Causality tests in extreme movements

	SPCA															PCA
δ	1,000	1,309	1,402	1,711	1,834	1,933	2,017	2,022	2,050	2,125	2,139	2,199	2,249	2,304	2,327	2,388
Id_k	Id_1	Id_2	Id_3	Id_4	Id_5	Id_6	Id_7	Id_8	Id_9	Id_{10}	Id_{11}	Id_{12}	Id_{13}	Id_{14}	Id_{15}	Id_{16}
k	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9	k=10	k=11	k=12	k=13	k=14	k=15	k=16
U(10)	2.91	1.13	1.13	4.99	3.83	3.83	3.83	3.83	3.83	4.43	4.43	5.95	5.95	**9.08**	6.83	6.83
U(25)	5.81	1.52	1.52	5.70	3.71	3.71	3.71	3.71	3.71	4.92	4.92	7.31	7.31	**11.22**	7.67	7.67

Note: Bloomberg, daily data from the 09/03/2003 to the 02/26/2016; authors' computation. The Table shows the value of the U(.) statistic of Hong et al. (2009) in Eq. (13) for inference on causality from monthly variations of each aggregate index to the monthly change in industrial production. Id_1 to Id_{16} correspond to the various aggregated indices of systemic risk. The threshold for significance at nominal risk level of 5% is 1.96.

The optimal aggregate index (that we are going to name the ISRM from now on) is thus, based on the results of Table 2, entirely determined by: the Default Yield Spread, the Term Spread, the Herfindahl-Hirschman Index, the Absorption Ratio by Kritzman et al. (2011), the Spillover Index by Diebold and Yilmaz (2009), the SRISK by Acharya et al. (2012) and Brownlees and Engle (2017), the Aggregated Vol, the MES of Acharya et al. (2010), the

Component Expected Shortfall of Banulescu and Dumitrescu (2015), the Value-at-Risk and the CoVaR and the ΔCoVaR of Adrian and Brunnermeier (2016), the Dynamical Causality Index of Billio et al. (2012) and the turbulence index of Kritzman and Li (2010).

The largest contributor to the aggregate index is the Spillover Index of Diebold and Yilmaz (2009) with a weight of .69; conversely, the one with the lowest impact is the Turbulence index of by Kritzman and Li (2010) with a weight of .03. Finally, three complementary dimensions of systemic risk are taken into account in our aggregate index: the liquidity (Amihud Illiquidity Measure), the contagion effect measure (the Spillover Index) and the concentration risk component (measured by the Herfindahl-Hirschman Index), in addition to the size of the institution and leverage effect encompassed in the SRISK.

In the following, we continue our analysis on the relationship between the GDP and the ISRM to see if the ISRM can explain future variations of the GDP. We also conduct the same analysis between the market index and the ISRM using the indexes obtained by SPCA and by PCA.

We start by computing quarterly series of the ISRM, the GDP and the market index (the S&P500 here). With Ordinary Least Squares (OLS) and Quantile Regression (QR), we estimate the relationship between the GDP/market index and the systemic risk index (lagged) from the Q1-06 to the Q4-15 following the equation:

$$\dot{y}_{t+1} = \mu + \lambda IS\dot{R}M_t + \xi_t, \tag{16}$$

where $\dot{y}_{t+1} = (y_{t+1} - y_t/y_t)$ is either the variation rate of the macroeconomic variable or later the variation rate of the market index, $IS\dot{R}M_t = (ISRM_t - ISRM_{t-1}/ISRM_{t-1})$is the variation rate of the systemic risk index and ξ_t is the residual at time t.

Figure 4 shows the dynamic of the R^2 for the relationship between the one ahead period GDP and systemic risk index (lagged) using Ordinary Least Squares (OLS) and Quantile Regression (QR) at the present period. The explicative power seems to increase during the financial crisis of 2008-2009. We can distinguish three change in regime periods along the entire sample. Here are plotted the two R^2 dynamics that represents in fact the two estimation methods (OLS) and (QR) for the index obtained by Sparse-Principal Component Analysis (SPCA).

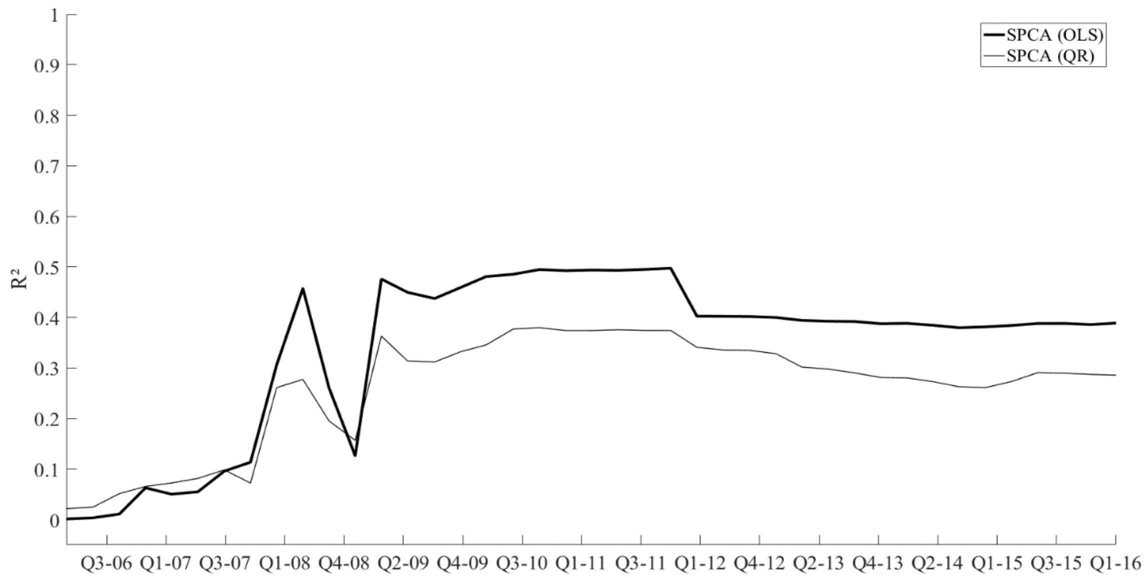

Fig. 4. R^2 dynamic for the GDP-ISRM relationship. Note: Bloomberg, quarterly GDP series from the Q1-06 to the Q4-15; authors' computation. The Figure shows the dynamic of the R^2 for the relationship between GDP and systemic risk index using ordinary least squares (OLS) and Quantile Regression (QR) with a 20th percentile for a given t+1 horizon (dynamic evolution of the link according to *t*);

We continue our analysis by looking for what is the better forecast horizon of the ISRM to predict changes in the GDP growth rate. Figure 5 represents the adjusted R^2 of the relationship between GDP and ISRM at the $t + h$ period ahead from 1 to 10 periods. Here, the R^2 seems to goes down slowly as the forecast horizon increases.

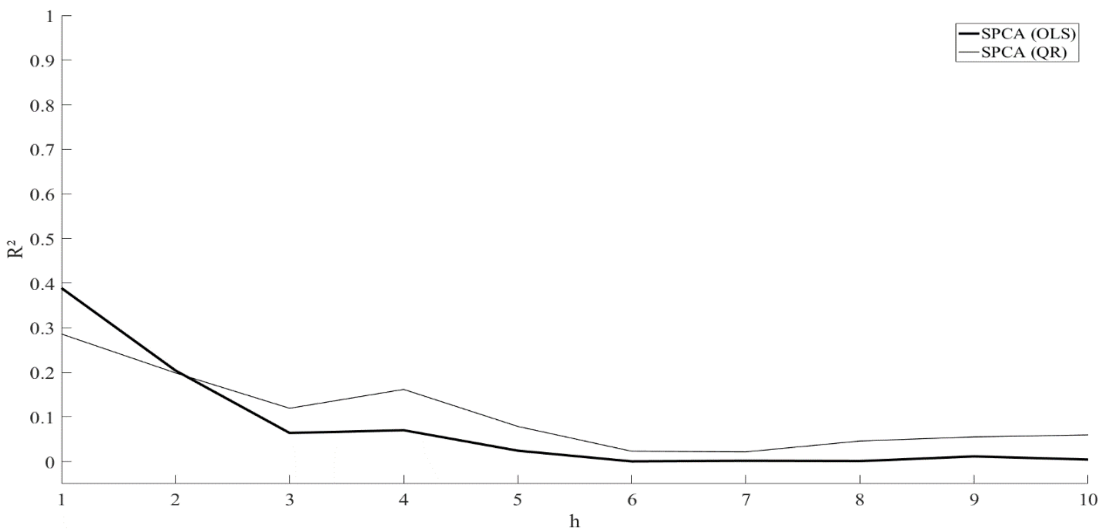

Fig. 5. R^2 dynamic for the GDP-ISRM relationship with respect to t+h period ahead. Note: Bloomberg, quarterly GDP series from the Q1-06 to the Q4-15; authors' computation. The Figure shows the dynamic of the R^2 for the relationship between GDP and systemic risk index using ordinary least squares (OLS) and Quantile Regression (QR) with a 20th percentile for a given t+h horizon (mean general relation according to h).

4. Robustness Tests

This section is devoted to the robustness check of the build method of the aggregated index based on systemic risk measures.

To check the robustness of our approach, we first compared the loadings obtained by the SPCA with the loadings obtained by a simple Ordinary Least Squares (OLS) regression of this component on matrix of systemic risk measures. We find also similar weights. However, regressing successively this component by adding one by one the explicative variable (in the order entry or not presented in Table 2), allows us to find similar weights again.

Furthermore, a backward regression based on Student statistics corrected for the coefficients attached to the regression variables, by adding (removing) the variables with respect to their significance levels gives similar coefficients and the same order of selection of the variables in the global indices.

In addition to the work of Kouontchou et al. (2017), we propose to test comparatively other similar methods to construct the aggregate index by considering two other types of similar penalized regressions known as "RIDGE" and "Elastic-net". Hastie et al. (2015) recall the properties of these regressions and compare them with the "LASSO" regression and its generalizations. The constraint or penalty of the so-called "RIDGE" regression is no longer based on the norm 1 as in the case of the "LASSO" regression, but on the norm 2 of the parameter vector β^s. Keeping the same target equation for the regression as in (5), the constraint changes into:

$$s.c. P(\beta^s) \equiv \|\beta^s\|_2 = \sum_{j=1}^{p} \left(\beta_j\right)^2 \leq \delta \,, \tag{17}$$

with $\|.\|_2$ the 2-norm.

The second penalized regression that we take into account is called "Elastic-net". This regression is a combination of the penalized regressions "LASSO" and "RIDGE" and, using the same objective regression as in (5), the constraint is now reading:

$$s.c. P(\beta^s) \equiv \frac{1-\alpha}{2} \|\beta^s\|_2 + \alpha \|\beta^s\|_1 = \sum_{j=1}^{p} \left[\frac{1-\alpha}{2} \left(\beta_j\right)^2 + \alpha \left|\beta_j^s\right|\right] \leq \delta \,, \tag{18}$$

with α the smoothing parameter to balance both penalties expressed in the norms 1 and 2 of the parameter vector β^s. In the extreme case where $\alpha = 1$, the penalty $P(\beta^s)$ is reduced to the constraint on the norm 1 of the parameter vector β^s, returning to the "LASSO" regression. In the other extreme case where $\alpha = 0$, the constraint $P(\beta^s)$ is just reduced on the constraint on the norm 2 of the parameter vector β^s which corresponds to the "RIDGE" regression.

We compare our results using successively these three penalized regressions, switching among the norm 1 (LASSO) to the norm 2 (RIDGE) and a combination of both (Elastic-net).

Table 3

Comparison of the Id_{14} indices according to three penalized regressions

	LASSO (norm 1)	Elastic-net (norms 1 and 2)	RIDGE (norm 2)
Indices	**Id_{14}**	**Id_{14}**	**Id_{14}**
δ	2.30	2.30	2.30
M_1	.6931	.6931	.6932
M_2	.5606	.5606	.5606
M_3	.0983	.0981	.0980
M_4	.4039	.4039	.4039
M_5	.0683	.0686	.0689
M_6	.0434	.0439	.0443
M_7	.0632	.0632	.0632
M_8	.0654	.0654	.0654
M_9	.0569	.0566	.0563
M_{10}	.0829	.0828	.0827
M_{11}	.0559	.0558	.0558
M_{12}	.0375	.0375	.0375
M_{13}	.0268	.0268	.0268
M_{14}	.0480	.0478	.0477
M_{15}	.0000	.0000	.0000
M_{16}	.0000	.0000	.0000
U(10)	9.0752	9.0752	9.0752
U(25)	11.2200	11.2200	11.2200

Note: Bloomberg, daily data from the 09/02/2003 to the 26/02/2016 for a set of 95 financial institutions; authors' computation. The table shows the comparison of the optimal index among the 3 approaches: LASSO, Elastic-net and RIDGE for the loadings and the statistic of Hong et al. (2009). The first line gives the value of the parameter δ. The 16 following lines give the loadings of the 16 measures M_1 to M_{16} in this order: M_1: Spillover Index, M_2: Herfindahl-Hirschman Index, M_3: VaR, M_4: Absorption Ratio, M_5: CoVaR, M_6: MES, M_7: Term Spread, M_8: Default Yield Spread, M_9: CES, M_{10}: Volatility, M_{11}: Dynamical Causality Index, M_{12}: SRISK, M_{13}: Turbulence Index, M_{14}: ΔCoVaR, M_{15}: TED Spread and M_{16}: Amihud Illiquidity Measure. The last two lines give the values of the statistic U of the Hong et al. (2009) for a parameter $d = 10$ and $d = 25$.

Given the results in this Table 3, these[8] are very similar whether in the composition of the optimal index or for the test statistic Hong et al. (2009). We conclude that the three approaches yield very similar results and confirm the optimal choice of the Id_{14} index. The same systemic

[8] As before, we want to know if the index is optimal for a set of values $d = 10$ to $d = 25$ according to the three penalized regressions. The results confirm the choice of the Id_{14} index for the three penalized regressions.

risk measures are active in the composition of the index with small differences in their loadings regarding the three approaches while the Hong et al. (2009) U-statistics are the same.

Table 4 provides the number of significant systemic risk measures based on the Kaiser criterion and also the rankings of the Id_k indices in terms of parsimony according to different information criteria: Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC) and Schwarz Information Criterion (SIC) and according to the U-statistic of the test of Hong et al. (2009) for the three penalized regressions: "LASSO", "Elastic-net" and "RIDGE". As expected, the ranking according to the U-statistic of the test of Hong et al. (2009) highlights the optimal index obtained previously regardless of the type of penalized regression used.

Table 4

Comparison of the Id_{14} indices according to various parsimony criterions

Id_k	Id_1	Id_2	Id_3	Id_4	Id_5	Id_6	Id_7	Id_8	Id_9	Id_{10}	Id_{11}	Id_{12}	Id_{13}	$\mathbf{Id_{14}}$	Id_{15}	Id_{16}
LASSO (norm 1)																
Kaiser	1	2	2	**4**	4	4	5	5	5	5	5	4	3	**3**	3	3
AIC	7	4	12	**1**	6	9	10	8	15	13	2	16	5	**11**	14	3
BIC	7	4	12	**1**	6	9	10	8	15	13	2	16	5	**11**	14	3
SIC	7	4	12	**1**	6	9	10	8	15	13	2	16	5	**11**	14	3
U(10)	6	15	16	**7**	10	11	12	13	14	8	9	4	5	**1**	2	3
U(25)	6	15	16	**7**	10	11	12	13	14	8	9	4	5	**1**	2	3
Elastic-net (norms 1 and 2)																
Kaiser	1	2	2	**4**	4	4	5	5	5	5	5	4	3	**3**	3	3
AIC	9	4	12	**1**	5	7	10	8	15	13	2	16	6	**11**	14	3
BIC	9	4	12	**1**	5	7	10	8	15	13	2	16	6	**11**	14	3
SIC	9	4	12	**1**	5	7	10	8	15	13	2	16	6	**11**	14	3
U(10)	6	15	16	**7**	10	11	12	13	14	8	9	4	5	**1**	2	3
U(25)	6	15	16	**7**	10	11	12	13	14	8	9	4	5	**1**	2	3
RIDGE (norm 2)																
Kaiser	1	2	2	**4**	4	4	5	5	5	5	5	4	3	**3**	3	3
AIC	10	4	12	**1**	6	9	7	8	15	13	2	16	5	**11**	14	3
BIC	10	4	12	**1**	6	9	7	8	15	13	2	16	5	**11**	14	3
SIC	10	4	12	**1**	6	9	7	8	15	13	2	16	5	**11**	14	3
U(10)	6	15	16	**7**	10	11	12	13	14	8	9	4	5	**1**	2	3
U(25)	6	15	16	**7**	10	11	12	13	14	8	9	4	5	**1**	2	3

Note: Bloomberg, daily data from the 09/02/2003 to the 26/02/2016 for a set of 95 financial institutions; authors' computation. The table shows the comparison of the optimal index among the 3 approaches: LASSO, Elastic-net and RIDGE. In each one of the three parts of the Table, the first line gives the number of components with respect to the Kaiser criterion, the next three lines give the ranking of the 16 indices according to the AIC, BIC and SIC criterions, At the end of each part, the last two lines give the values of the U statistic of the Hong et al. (2009) test for a parameter $d = 10$ and $d = 25$. Bold values indicate the best choice for each criterion and the values for the optimal index according to the Hong et al. (2009) test.

The number of measures respecting the Kaiser criteria in the optimal index (according to the parsimony criterions AIC, BIC and SIC) is 4, highlighting the importance of the following measures in the construction of the index: M_1: Spillover Index, M_2: Herfindahl-Hirschman Index, and M_4: Absorption Ratio which takes increasing importance at the expense of the M_3 measure corresponding to the VaR which gradually loses importance in the composition of the index when the number of active measures increases, reflecting a selection in the variability of our set of measures in order to take into account the different aspects of systemic risk since the risk of extreme loss linked to the definition of the VaR is also found in other measures present in the optimal index.

The parsimony criterions confirm this aspect in terms of index rankings. The optimal index according to the AIC, BIC and SIC criteria is Id_4 while the least parsimonious is Id_{12}. Thus, the criterion of Hong et al. (2009) tends to favor a less parsimonious index (Id_{14}), whereas the usual information criteria would lead us (all unanimously moreover) to consider simpler models (such as Id_4 or Id_{11}).

Moreover, taking into account the aggregation of all the main components as the target of the penalized regression gives similar results in terms of weightings in the composition of the index, in terms of values of Hong et al. (2009) test and in terms of the choice of the model and the variables to be retained which are identical.

Conclusions

This article presents an aggregated measure of global systemic risk. The rationality of the exercise lies in the multiplicity of systemic risk indicators introduced in the literature since the last global financial crisis, and the discrepancies between them. The latter arises from the fact that each metric evaluates a particular facet of systemic risk, but also is due to the model risk inherent in their estimation.

The proposed modelling methodology is based on the Sparse Principal Component Analysis (SPCA), which, in contrast to conventional PCA, allows us to select a reduced number of systemic risk measures for the construction of the aggregate index. As a result, the obtained index is more parsimonious and has, by construction, a more stable dynamic over time.

From the results obtained using US data, it appears that the optimal index thus created is in fact less volatile than that based on the traditional PCA used in previous studies. Moreover, it appears that positive extreme movements of the index are advanced predictors of a sudden severe slowdown in economic activity.

In terms of our main object of study - the overall systemic risk, we can see from the common characteristics of 1) the overall evolution of the various measures of systemic risk, 2) that of the indices, and 3) that of the optimal index proposed in this article: a strong upward trend in systemic global risk prior to the 2007-2008 crisis (as a precursor to a growing risk phenomenon, the result of which will be a major financial crisis and affecting the real economy), a peak in late 2008, before a relatively rapid decline at the end of the crisis, leaving place to some stabilization of total risk in recent years but still at a higher level than the crisis.

Several extensions of this work can be envisaged. First, the use of an out-of-sample analysis (although difficult given the available samples) could be used to confirm the results obtained in sample. Secondly, applying the methodology to other (European) data would make it possible to assess the robustness of the US results and the differences in responses to the latest crisis periods in Europe (Cf. Ben Bouheni and Hasnaoui, 2017). Finally, our methodology does not prevent to take into account of new risk measures. Finally, a regular monitoring of the index of systemic risk measures would undoubtedly help to assess market conditions and the dynamics of the overall systemic risk.

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Appendices

Appendix A: The main systemic risk measures

This appendix presents the details of the various systemic risk measures used in this study except for the macro financial variables. For the individual systemic risk measures indexed by i, a global measure is derived: it corresponds to the cross sectional mean of the individual measures at time t. The description order is deduced here by the entry order of the systemic risk measures in the composition of the index when using the SPCA (see Table 2). The measures: M_7: Term Spread, M_8: Credit Default Yield Spread and M_{15}: TED Spread complete the listed ones below.

- **Measure 1 (M_1), Spillover Index *(SI)***: this index proposed by Diebold and Yilmaz (2009), aggregates the contribution of each variable to the error total variance forecast on the returns. It quantifies the degree of contagion in the system and is expressed as a VAR of order p for N variables to forecast H periods ahead as follow:

$$SI_t = \frac{\sum_{h=0}^{H-1} \sum_{\substack{i,j=1 \\ i \neq j}}^{N} a_{h,ij}^2}{\sum_{h=0}^{H-1} trace(A_h A_h')} \times 100 \,, \tag{A1}$$

with A_h the matrix of contributions of each financial institution from their returns in the VAR(p) framework with the A_h element denoted $a_{h,ij}$ is the contribution to the variance of the institution i by the institution j. The numerator is the sum of these contributions on each forecast periods ahead represents the total spillover in the system. The denominator corresponds to the total variance of the forecast error; as an illustration, for $h = 0$ and $i, j = 1,2$, then $a_{0,11}^2 + a_{0,12}^2 + a_{0n21}^2 + a_{0,22}^2 = trace(A_0 A_0')$.

- **Measure 2 (M_2), Herfindahl-Hirschman Index *(HHI)***: is an index quantifying the concentration in the system. It captures the potential fragility of the system from its concentration and the threat of the defaults of the largest companies. It is defined as the sum of the squared market values out of the squared sum of these same market values such as:

$$HHI_t = N \frac{\sum_{i=1}^{N} \left(ME_{i,t}\right)^2}{\left(\sum_{i=1}^{N} ME_{i,t}\right)^2} \,, \tag{A2}$$

with $ME_{i,t}$ the market value of the institution i at time t and N the number of institutions.

- **Measure 3 (M_3), Value-at-Risk (*VaR*)**: is the Value-at-Risk of the system or for a market index. It is the maximal potential loss for a given probability on a time horizon and is defined as:

$$\Pr\left(r_{i,t} \leq VaR_{i,t}(\alpha)\right) = \alpha \,, \tag{A3}$$

with $r_{i,t}$ returns of the institution i at time t for a given risk level α.

- **Measure 4 (M_4), Absorption Ratio *(AR)*:** proposed by Kritzman et al. (2011), it measures the tendency of the markets to co-move in the same way and is written as follow:

$$AR_t = \frac{\sum_{j=1}^{J} \sigma_{E_{j,t}}^2}{\sum_{i=1}^{N} \sigma_{a_{i},t}^2}, \tag{A4}$$

with J the number of eigen vectors, $\sigma_{E_{j,t}}^2$ the variance of the eigen vector j and $\sigma_{a_{i,t}}^2$ the variance of the asset i at time t. The Eigen values and vectors are obtained from the variance covariance matrix (VCV) of the N asset returns at time t. Only the J largest eigen values are summed to get the numerator while the denominator is the $trace$ of the VCV matrix.

- **Measure 5 (M_5), Conditional Value-at-Risk (*CoVaR*)**: introduced by Adrian and Brunnermeier (2016), it corresponds to the VaR of the system (or simply the VaR of the market index), conditional on institutions being under distress. If one denotes $r_{m,t}$ the market returns, then we have:

$$\Pr\left(r_{m,t} \leq CoVaR_{i,t}(\alpha) \middle| r_{i,t} \leq VaR_{i,t}(\alpha)\right) = \alpha, \tag{A5}$$

with $r_{i,t}$, the returns of the institution i and $CoVaR_{i,t}(\alpha)$ the *CoVaR* of the institution i at time t for a given risk level α.

- **Measure 6 (M_6), Marginal Expected Shortfall (*MES*)**: is proposed by Acharya et al. (2013) and (2016); it is defined as the conditional mean returns of the institution i when the market, as a whole, is in distress. This situation can be written as follow:

$$MES_{i,t}\left(\alpha = E\left(r_{i,t} \middle| r_{m,t} \leq VaR_{m,t}(\alpha)\right)\right), \tag{A6}$$

with $r_{i,t}$, $r_{m,t}$are the returns of the institution i and the returns of the market and $VaR_{m,t}(\alpha)$ is the VaR of the market portfolio for a given risk level α, at time t. The MES is equal to the partial derivative of the Expected Shortfall (ES) of the market portfolio with respect to the weights of the institution i, and then measures its marginal systemic risk contribution.

- **Measure 9 (M_9), Component Expected Shortfall (*CES*)**: is introduced by Banulescu and Dumitrescu (2015); it quantifies the contribution of an institution to the risk of the system by multiplying the $MES_{i,t}$ of this institution at time t by its weight in the system such as:

$$CES_{i,t}(\alpha) = -w_{i,t} MES_{i,t}\left(\alpha = E\left(r_{i,t} \middle| r_{m,t} \leq VaR_{m,t}(\alpha)\right)\right), \tag{A7}$$

where $r_{i,t}$, $r_{m,t}$ are the returns of the institution i and the returns of the market and $VaR_{m,t}(\alpha)$ is the VaR of the market portfolio for a given risk level α, at time t.The weight of the institution i denoted $w_{i,t}$ is simply its market value divided by the total market value of the system.

- **Measure 10 (M_{10}), Volatility *(Vol)***: is the aggregated volatility of all the financial institutions in the system or simply the volatility of a market index. It is defined as the standard deviation of a one year period of opening days.

- **Measure 11 (M_{11}), Dynamic Causality Index *(DCI)***: built by Billio et al. (2012); it measures the degree of interconnexion in the system as the number of significant Granger causalities dived by the total number of Granger causalities such as:

$$DCI_t = (\#GC_t^*)/(\#GC_t), \tag{A8}$$

with $\#GC_t^*$ the number of significant Granger causalities and $\#GC_t$ the total number of Granger causalities at time t.

- **Measure 12 (M_{12}), Systemic RISK (*SRISK*)**: proposed by Acharya et al. (2012) and Brownlees and Engle (2017); it corresponds to the amount of capital needed by a firm in distress when the market is also in distress and is defined as:

$$SRISK_{i,t}(1-\alpha) = \max\{0, \gamma D_{i,t} - (1-\gamma)W_{i,t}[1 - LRMES_{i,t}(1-\alpha)]\}, \tag{A9}$$

with γ the *prudential capital requirement* required by the regulator**,** $D_{i,t}$ the amount of debt and $W_{i,t}$ the amount of liabilities of the institution i at time t. $LRMES_{i,t}(1-\alpha)$ is the long run approximation (six months) of the $MES_{i,t}(1-\alpha)$ of the institution i at time t and is defined such as:

$$LRMES_{i,t}(1-\alpha) \approx 1 - exp[18 \times MES_{i,t}(1-\alpha)]. \tag{A10}$$

- **Measure 13 (M_{13}), Turbulence Index *(TI)***: introduced by Kritzman and Li (2010), this index reflects the excess volatility and compares the squared realized returns to their historical volatility; it is written:

$$TI_t = (r_t - \mu)'\Sigma^{-1}(r_t - \mu), \tag{A11}$$

with r_t the vector of the returns, μ the historical mean returns and Σ the variance covariance matrix of the returns.

- **Measure 14 (M_{14}), Delta Conditional Value-at-Risk (*ΔCoVaR*)**: also proposed by Adrian and Brunnermeier (2016); it is the difference between the *CoVaR* of the institution i at a given risk level $\alpha = 5\%$ and the *CoVaR* of the same institution but at $\alpha = 50\%$ (median state).

$$\Delta CoVaR_{i,t}(\alpha) = CoVaR_{i,t}(\alpha) - CoVaR_{i,t}(0.5), \tag{A12}$$

where $CoVaR_{i,t}(\alpha)$ is the VaR of the system for a given risk level α, conditional on institution i being under distress at time t.

- **Measure 16 (M_{16}), Amihud Illiquidity Measure (*AIM*):** built by Amihud (2002), this measure captures the illiquidity level of the trades on a given asset. The variable $AIM_{i,t}$ is defined:

$$AIM_{i,t} = \frac{1}{K} \sum_{\tau=t-K}^{t} \frac{\left| r_{i,\tau} \right|}{TO_{i,\tau}}, \tag{A13}$$

where $\left| r_{i,\tau} \right|$ is the absolute return of the institution i and $TO_{i,\tau}$ is the turnover of the same asset at time τ, on a given period from $t - K$ to t.

Then, the three financial macro variables used generally as leading indicators of economic activity (see Estrella and Trubin, 2006; Chen et al., 2009) must be added to this list of measures:

- **Measure 7 (M_7), Term Spread** which measures the slope of the yield curve and which corresponds to the yield spread between 10-year and 3-month Treasury bills - this variable is a leading indicator of economic activity (Estrella and Trubin, 2006);
- **Measure 8 (M_8), Credit Default Yield Spread** which represents the difference between the yield of corporate bonds rated BAA and the ones rated AAA by Moody's; Chen et al. (2009) show that this variable is an aggregated measure of the robust credit risk to frictions (tax and liquidity) on the bond market;
- **Measure 15 (M_{15}), TED Spread** which represents the difference between the LIBOR three-month rate and sovereign interest rates to three months - an increase of this variable is the sign that lenders expect an increase in credit risk in the interbank lending market.

Appendix B: The financial institutions

Are presented below, the financial institutions that served to build the systemic risk measures (see Brownlees and Engle, 2017). These institutions fall into four financial industry group covering the depositories, the insurers, the broker-Dealers agencies. The last one group is dedicated to the "Others" financial institutions that are not in the other three groups.

Table A1

Tickers of Financial Institutions

Banks			**Insurances**	
Code	**Institution**		**Code**	**Institution**
BAC	Bank of America		ABK	Ambac Financial Group
BK	Bank of New York Mellon		AIG	American International Group
BBT	BB&T		CB	Chubb Corp.
C	Citigroup		CNA	CNA Financial Corp.
HBAN	Huntington Bancshares		CVH	Coventry Health Care
JPM	JP Morgan Chase		HUM	Humana
MTB	M&T Bank Corp.		LNC	Lincoln National
MI	Marshall & Ilsley		MMC	Marsh & McLennan
NTRS	Northern Trust		MBI	MBIA
SOV	Sovereign Bancorp		PGR	Progressive
STT	State Street		TRV	Travelers
SNV	Synovus Financial		UNH	UnitedHealth Group
Other financial institutions			**Brokers**	
Code	**Institution**		**Code**	**Institution**
ACAS	American Capital		ETFC	E*Trade Financial
AXP	American Express		GS	Goldman Sachs
COF	Capital One Financial		LEH	Lehman Brothers
EV	Eaton Vance		MER	Merill Lynch
FITB	Fifth Third Bancorp		MS	Morgan Stanley
BEN	Franklin Resources		SCHW	Schwab Charles
LM	Legg Mason		TROW	T. Rowe Price
SEIC	SEI Investment Company			
SLM	SLM Corp.			
AMTD	TD Ameritrade			

Note: Bloomberg. Cylindrical sample of 60 American financial institutions (Cf. Brownlees and Engle, 2017).

Note: This Table reports the tickers of the institutions.

Appendix C: Correlations among the systemic risk measures

Table A2

Pearson and Spearman correlations among the systemic risk measures

	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9	M_{10}	M_{11}	M_{12}	M_{13}	M_{14}	M_{15}	M_{16}
M_1	**1,00**	.43	.59	**.84**	.60	.57	.45	.61	.58	.61	-.23	**.84**	.23	.66	.26	.61
M_2	.40	**1,00**	.38	.41	.43	.35	.68	.39	.33	.51	.03	.47	-.11	.35	-.19	.60
M_3	.79	.36	**1,00**	.37	**.94**	**.97**	.31	**.87**	**.97**	.79	.11	.59	.52	**.93**	.67	.57
M_4	.72	.30	.48	**1,00**	.31	.36	.54	.34	.37	.37	-.24	**.83**	.09	.42	.06	.45
M_5	.67	.35	.77	.23	**1,00**	**.92**	.33	**.89**	**.91**	**.82**	.17	.50	.45	**.94**	.57	.62
M_6	.72	.26	**.91**	.49	.71	**1,00**	.24	**.88**	1,00	.72	.13	.60	.52	**.94**	.69	.49
M_7	.32	.63	.44	.33	.38	.26	**1,00**	.18	.24	.47	-.01	.44	-.09	.25	-.11	.54
M_8	.71	.29	.70	.32	.68	.66	.08	**1,00**	**.87**	.74	.09	.56	.40	**.85**	.55	.52
M_9	.73	.25	**.89**	.49	.71	**.99**	.25	.66	**1,00**	.70	.13	.61	.53	**.94**	.69	.48
M_{10}	.76	.47	**.89**	.48	.67	.74	.61	.65	.72	**1,00**	-.07	.53	.16	.64	.28	**.93**
M_{11}	-.21	.02	-.23	-.17	.01	-.13	-.05	-.21	-.11	-.32	**1,00**	-.31	.20	.14	.21	-.22
M_{12}	.75	.38	.76	.67	.40	.74	.28	.61	.74	.74	-.36	**1,00**	.15	.59	.16	.56
M_{13}	.14	-.24	.16	.05	.22	.23	-.28	.17	.22	-.05	.25	.02	**1,00**	.56	.65	-.05
M_{14}	**.81**	.33	**.82**	.55	**.85**	**.81**	.34	.67	**.82**	.67	-.02	.62	.27	**1,00**	.68	.43
M_{15}	.08	-.39	.15	-.19	.34	.26	-.49	.41	.26	-.03	.17	-.01	.48	.25	**1,00**	.00
M_{16}	.76	.64	.75	.64	.46	.61	.63	.54	.60	**.88**	-.39	**.84**	-.20	.54	-.29	**1,00**

Note: Bloomberg, daily data from the 09/03/2003 to the 02/26/2016; authors' computation. M_1 to M_{16} represent the 16 systemic risk measures and are in the following order: M_1: Spillover Index, M_2: Herfindahl-Hirschman Index, M_3: VaR, M_4: Absorption Ratio, M_5: CoVaR, M_6: MES, M_7: Term Spread, M_8: Default Yield Spread, M_9: CES, M_{10}: Volatility, M_{11}: Dynamical Causality Index, M_{12}: SRISK, M_{13}: Turbulence Index, M_{14}: ΔCoVaR, M_{15}: TED Spread and M_{16}: Amihud Illiquidity Measure. The significant correlation at 5% level are in black, the non-significant correlations are in grey and the correlations above .80 are in bold. The out-of-diagonal elements (upper top-right) are the Pearson correlations, while the others out-of-diagonal elements (lower bottom-left) are the Spearman correlations. When computing the correlations for the conditional concordant extreme variations of measures above their respective 20th quantiles, they appear to be all equal to 1.