Optimal Procurement with Take-or-Pay Contracts in the Presence of Storage

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In this paper, we study a procurement decision model for a take-or-pay contract in the presence of spot market trading and storage capability in a dynamic setting using multi-stage stochastic programming. Designing a computationally efficient split-variable formulation, we solve the procurement problem and delineate the impact of key managerial levers such as take-or-pay contract penalty cost and percentage obligation, net convenience yield, and storage capacity on the take-or-pay contract design, valuation, and usage. Furthermore, we numerically show the sub-additivity of option value for take-or-pay and storage options, which is highly important in option portfolio construction. Practically relevant managerial insights are generated to assist decision makers in commodity supply chains to better design, plan and act where spot market price and storage uncertainties abound.

Key words : supply chain risk; stochastic programming; commodity markets

1. Introduction

Today's volatile energy markets require more flexibility in supply contract management. Especially, natural gas and LNG markets that operate with long term take-or-pay supply contracts would appreciate not only price, but also volume flexibility. These concerns have been constantly voiced by policy and decision makers in the energy industry not only in US or Europe but also Asia, Latin America, and the emerging markets. How is such flexibility factored into managerial decisions? What managerial levers can one use in managing supply and demand risks as well as storage and price risks along the natural gas supply chains? These are some of the critical questions pondered on constantly by decision makers.

In such an uncertain and complex world, it is interesting to see that buyers of natural gas are tied into contracts that can last for decades. For instance, *BOTAŞ*, *Petroleum Pipeline*

Corporation of Turkey, procures natural gas via Blue Stream Project from Russia in a 25-year contract. Having longer duration contracts surely encourage these pipelines to be built and operated in the first place. Steady stream of procurement is necessary for the seller to invest in gas production and transportation. However, due to various uncertainties in the market such as weather risk, climate change, and alternative energy sources such as renewables, end consumer demand becomes more volatile. Thus, the use of natural gas in power generation as well as heating could be lower than previously envisaged. In such cases, buyers such as BOTAŞ would be willing to renegotiate the volumes they have to procure via the contract as well as the contract prices.

In this paper, we argue that take-or-pay contracts would create more value in the market if the buyer can use the spot market purchasing as well as store the natural gas for future consumption even if there is no immediate demand in the market. Thus, it is critical for a natural gas buyer that is locked into a take-or-pay contract to consider the procurement from the spot market as well as the option for storage in order to minimize its total procurement costs during a long-term relationship.

Therefore, with the strong motivation from the natural gas market, we aim to solve the buyer's procurement problem that is locked into a long term take-or-pay contract, yet has the option to intelligently use spot market purchasing and the storage.

The remainder of the paper is organized as follows. Section 2 summarizes the relevant previous work and positions our paper in the literature. Section 3 presents the model, two alternative formulations of the multi-stage stochastic program, and some structural results. Section 4 provides the computational study with key managerial insights. Section 5 concludes with discussion and future research directions.

2. Related Literature

Our work is related to two separate streams of literature. One is the work in supply chain contracts and energy/commodity procurement. The second line of work is the financial and real options valuation for multiple, interacting options.

In supply chain contract and energy procurement literature, the first branch of research focuses on spot market trading and storage in commodity procurement and have been addressed recently by a number of scholars such as Haksöz and Seshadri (2007), Thompson et al. (2009), Secomandi (2010a, 2010b), Devalkar et al. (2011), Goel and Gutierrez (2011), Lai et al. (2010, 2011), Wu et al. (2012), and Secomandi and Kekre (2014). This line of work does not consider take-or-pay contracts and its intricacies of which our paper examines.

On the second branch of work, take-or-pay contracts in the energy industry have been addressed by Thompson (1995), Schultz (1997), Creti and Villeneuve (2004), Rodriguez (2008), Glachant and Hallack (2009), Wahab and Lee (2011). These papers study the structure of take-or-pay contracts in different settings with various goals such as examining the value of make-up provisions, contract robustness, and free destination flexibility. However these works study neither the storage option nor the spot market trading explicitly in the long term dynamics of this type of contract. As an exception, Kaiser and Tumma (2004) present a Monte Carlo simulation for binomial lattice valuation of a take-or-pay contract for ethylene that addresses the storage and spot price uncertainty. Within this branch, there have also been several attempts to employ numerical methods such as stochastic programming (Haarbrücker and Kuhn, 2009 and Pflug and Broussev, 2009), Monte-Carlo simulation (Ibáñez, 2004), and binomial forests (Jaillet et al., 2004).

In financial and real options literature, the most relevant papers to our work are in the area of the valuation of interacting options. In his seminal work, Trigeorgis (1993) studies the interactions among multiple real options such as deferral, abandonment, contraction, expansion, and switch. Option interactions may increase/decrease (super-additive versus sub-additive property is shown) the bundled value of the options. Agliardi (2006, 2007) demonstrate the value interactions analytically for two (expansion or contraction) options. Agliardi (2007) further shows that addressing more than two options is technically doable and creates more flexibility, however the mathematical complexity increases due to working with nested multinomial cumulative functions. Koussis et al., (2007) tackles the pricing problem of sequentially bundled real options utilizing the path-dependent claims approach in the take-or-pay contract literature. One recent work by Haksöz and Simsek (2010) in supply chain procurement literature models the bundled option value (abandonment and price renegotiation options) in the presence of spot market. Our paper is intimately related to the bundled option valuation due to having the spot market trading and the storage options on top of the take-or-pay contract.

Given the nature of complexity, we utilize multi-stage stochastic programming (MSP) to solve the pricing problem (Birge and Louveaux, 1997). MSP is a well-established approach for decision making under uncertainty especially in application areas such as finance and energy (Mulvey et al., 2008 and Gassmann and Ziemba, 2013).

Our contribution to the literature can be summarized as follows. We provide a procurement decision model that amalgamates the aforementioned disparate streams of literature for a take-or-pay contract with explicitly examined bundled options (spot market trading and storage) in a dynamic setting.

3. The Model

We model the dynamic procurement decision of a risk-neutral natural gas buyer who has the alternatives of purchasing via a take-or-pay contract or from the spot market as well as utilizing a temporal storage while minimizing the expected procurement cost in a multi-period setting. The buyer uses a standard take-or-pay contract with a single supplier. In this contract, the buyer pays the fixed (but not necessarily constant) contract price every period for the quantity purchased.

Similar to the previous literature, fixed contract prices are assumed to be the forward contract prices in our model, which are computed and known *ex ante*. We denote the forward contract price set at time 0 for a purchase at time period n as $P_{0.n}$. It is a function of the spot market price at time 0 and is computed as follows:

$$P_{0,n} = P_0 e^{(r-\delta)n\Delta t} \tag{1}$$

where P_0 is the spot market price at time 0, Δt is the length of one time period (in years), r is the constant annualized risk-free interest rate, and δ is the constant annualized convenience yield net of supplier's implied storage costs. We assume that the spot market prices are stochastic with a general known distribution.

Net convenience yield, which can be roughly defined as the difference between the future value of the spot price and the futures price based on (1), can always be observed and computed through market prices. This can be contrasted with the marginal convenience yield, which is the economic cost of holding a traded commodity in inventory and includes the cost of storage (Pindyck, 2004). Marginal convenience yield cannot be directly observed and, due to no-arbitrage, is always nonnegative. Net convenience yield, on the other hand, can become negative if the storage costs are relatively higher than the benefits of holding the commodity.

Throughout this paper, we utilize this latter definition of convenience yield and analyze its impact on the contract price.

Since the procurement contract is structured with a take-or-pay clause, the buyer has two decisions: i) how much it should procure at the fixed contract prices in period n, q_n^C, ii) how much it should procure from the spot market in period n, q_n^M. While making these decisions, the buyer is constrained by the maximum quantity to be procured via the contract. Moreover, if the buyer does procure less than the obligation of the take-or-pay contract, a penalty cost needs to be paid. We denote the remaining contractual obligation of the buyer by q_n^{ToP}, where $q_n^{ToP} \in \mathbb{R}$ at any time period. The penalty cost paid per unit quantity for the contractual obligation not taken is denoted by b, $b > 0$.

On the other hand, as opposed to the majority of the previous literature, we incorporate an important real-life consideration of storage possibility, where the natural gas can be stored by the buyer physically each period during the contract life. That is, the buyer not only procures the natural gas for consumption, but also can procure and store for future consumptions. Specifically, in each period, the buyer can store s_n amount where $s_n \in \mathbb{R}_+$. The buyer also needs to satisfy the deterministic demand in period n, denoted by d_n, where $d_n \in \mathbb{R}_+$.

Without the consideration of storage, the problem is equivalent to that in Thompson (1995) for unit demand per period and therefore can be solved easily through a dynamic programming (DP) setup as the only state variable is the remaining contractual obligation, q_n^{ToP}. However, the presence of storage complicates the problem by introducing a second state variable, the level of storage, s_n. As a result, when storage is costly and limited, DP formulation is no longer tractable. For the case of costless and unlimited storage, we analyze the value functions in detail and present them in the Appendix, where the aim is to understand the functional form in terms of the state variables.

3.1. MSP Model for Costless and Uncapacitated Storage

In this section, we assume that the buyer can store the natural gas it procures without incurring extra storage cost. This assumption is valid when the storage capacity investment has been already made and thus the cost is sunk. Moreover, the natural gas pipeline network needs to be also arranged such that storage is possible during the contract life. Second, we assume that the storage capacity is unlimited. This assumption will be relaxed later.

We model the contract pricing problem as a multi-stage stochastic program that minimizes the expected cost of procuring natural gas through the spot market or the take-or-pay contract while satisfying the demand. Costless and unlimited storage will not impose an extra constraint or a penalty, but it can possibly lead to a less expensive contract value due to the possibility of carrying inventory over time, increasing the temporal complexity of the problem.

In a multi-stage stochastic program, the uncertainty is typically represented through a scenario tree. In our case, this representation will apply to the spot market price of natural gas. A simple three-period (four-stage) trinomial scenario tree is illustrated in Figure 1. In this vertically drawn scenario tree where time progresses in the down direction, node 0 at the top represents the current state at time 0 and every other node is a different state of the world at the time points labeled on the right. For every step into the future from a given state, there are three possible states. Each path connecting the current state to a final state corresponds to a scenario. For example, in scenario 4, we observe nodes 1, 5, and 16 in the three periods. As a result, there are 40 nodes in total and 27 scenarios. It should be noted that a scenario tree can have any discrete-time and discrete-state structure in the form of uneven periodicity and unequal branching.

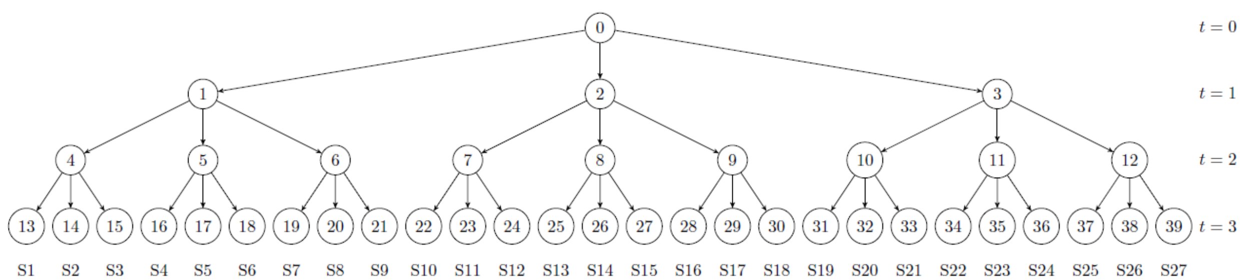

Figure 1 **Layout for a three-period scenario tree with three branches per node**

In MSP, this structure not only corresponds to the uncertainty modeling, but also displays the decision-making dynamics as well. In other words, at node 0, one decision is made with the knowledge of the full distribution represented by 27 scenarios. In the second stage ($t = 1$), three alternative decisions are possible in nodes 1, 2, and 3, depending on the realization of the first period. These decisions are conditional on the first-stage decision and are made with the knowledge of the current state and future distribution (9 scenarios in each case).

One can formulate a multi-stage stochastic program on a scenario tree in two ways. The first method, called the *compact formulation*, explicitly focuses on nodes on the tree, where decision variables and parameters are defined explicitly for each node. In the *split-variable formulation*, the deterministic problem is replicated for each scenario, regardless of the time stage. For a scenario tree as in Figure 1, this would mean having 27 variables for each stage on the tree. Clearly, this creates a perfect foresight issue for early stages, as we do not actually know what scenario we are in until the final stage. This is addressed by adding the so-called non-anticipativity constraints, which sets these early-stage variables equal to each other if they correspond to the same node on the tree. For our sample tree, this would mean that node 1, encompassing scenarios 1 to 9, will have 9 decision variables, but they are all set equal to each other with 8 non-anticipativity constraints.

3.1.1. Compact Formulation. For a scenario tree with K nodes (excluding the initial node labeled as zero) and N periods (i.e., $N+1$ stages), we define the following decision variables and parameters in addition to those defined earlier:

Parameters

P_k Spot price in node k

π_k Probability of node k

f_n Discount factor for a cash flow in n periods $\left(= e^{-rn\Delta t}\right)$

t_k Time stage of node k

$a_{n,k}$ Ancestor of node k in time stage n for $n \leq t_k$ ($a_{t_k,k} = k$)

Variables

q_k^M Amount procured in the spot market in node k

q_k^C Amount procured via the take-or-pay contract in node k

s_k Level of the storage in node k

The multi-stage stochastic programming model for the price of the contract can be formulated as follows:

$$\textbf{MSP1} \qquad \min \quad \sum_{k=1}^{K} \pi_k \times f_{t_k} \times \left(P_{0,t_k} q_k^C + P_k q_k^M \right) + \sum_{\forall k: t_k = N} \pi_k \times f_N \times b \left(q^{ToP} - \sum_{n=1}^{N} q_{a_{n,k}}^C \right)^+ \tag{2}$$

subject to

$$s_{a_{t_k-1,k}} + q_k^C + q_k^M = s_k + d_{t_k} \qquad k = 1,\ldots,K \tag{3}$$

In addition to the standard non-negativity constraints required for all s, q^C, and q^M variables, we set $s_0 = 0$ and $s_{k:t_k=N} = 0$ so that the initial and final storage levels are all zero. Ignoring the non-negativity constraints and these zero-level variables, for the scenario tree in Figure 1, we would have 90 variables and 39 constraints with the above formulation. Given the equality constraints, one could do better by explicitly replacing either q^C or q^M variables in the objective function with their equal parts and converting their non-negativity constraints to standard inequality constraints. This would reduce the number of variables but increase the number of constraints by the number of nodes. This modified formulation is left out due to space constraints; however we note that the compact form is not our preferred method although it may result in the lowest number of variables and constraints. The split-variable formulation (section 3.1.2) requires a larger number of variables and constraints; however, the resulting sparse A-matrix can be exploited by decomposition and interior-point algorithms for much faster solution times.

3.1.2. Split-Variable Formulation. For a scenario tree with J scenarios and N periods (i.e., $N+1$ stages), we define the following decision variables and parameters:

Parameters

$P_{n,j}$ Spot price at time point n in scenario j

π_j Probability of scenario j

$a_{n,j}$ Ancestor node for scenario j at time point n

Variables

$q_{n,j}^M$ Amount procured in the spot market at time point n in scenario j

$q_{n,j}^C$ Amount procured via the take-or-pay contract at time point n in scenario j

$s_{n,j}$ Level of the storage at time point n in scenario j

We can formulate the split-variable MSP as follows:

MSP2 $$\min \quad \sum_{j=1}^{J} \pi_j \left(\sum_{n=1}^{N} f_n \left(P_{0,n} q_{n,j}^{C} + P_{n,j} q_{n,j}^{M} \right) + f_N \left(b \left(q^{ToP} - \sum_{n=1}^{N} q_{n,j}^{C} \right)^{+} \right) \right) \tag{4}$$

subject to

$$s_{n-1,j} + q_{n,j}^{C} + q_{n,j}^{M} = s_{n,j} + d_n \qquad n = 1,\ldots,N; \quad j = 1,\ldots,J \tag{5}$$

$$q_{n,j}^{C} = q_{n,j+1}^{C} \qquad n = 1,\ldots,N; \quad j = 1,\ldots,J-1 : a_{n,j} = a_{n,j+1} \tag{6}$$

$$q_{n,j}^{M} = q_{n,j+1}^{M} \qquad n = 1,\ldots,N; \quad j = 1,\ldots,J-1 : a_{n,j} = a_{n,j+1} \tag{7}$$

In addition to the standard non-negativity constraints, we also set $s_{0,j} = s_{N,j} = 0$ for all scenarios to be consistent with the original model. Via the non-anticipativity constraints (6) and (7), the variables that are on the same node on the scenario tree but belong to different scenarios due to splitting take the same values. This is managed by checking the ancestors of scenarios on a scenario tree, as controlled by the parameter $a_{n,j}$.

To compare the size of the problem in this formulation with the one in the previous section, let us focus on the three-period case, where the trinomial model results in a set of 27 scenarios. In this case, we would have 216 variables and 81 constraints, excluding the non-anticipativity constraints. Clearly, this is a much larger constraint matrix compared to that of the compact formulation. The sparsity structure of this matrix is shown in Figure 2.

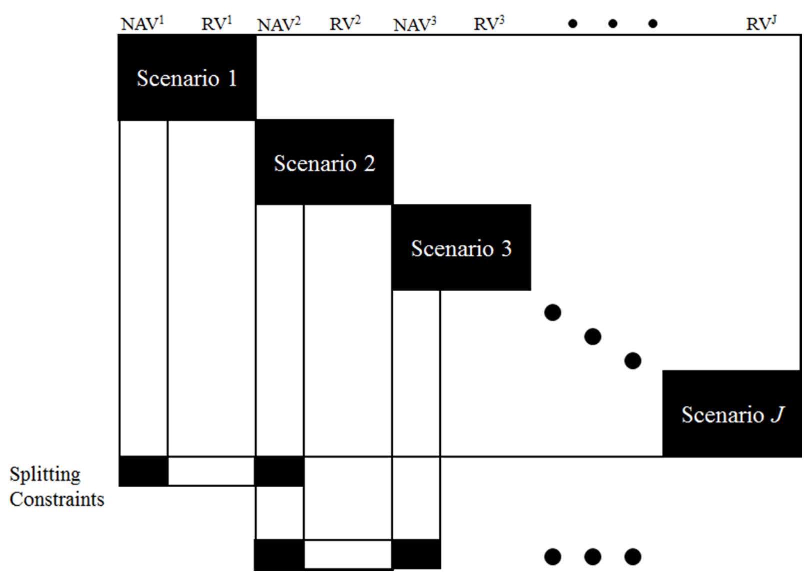

Figure 2 **Constraint matrix structure in the split-variable formulation**

The split-variable model has a significant technical advantage due to the fact that the non-anticipativity constraints are the only ones linking the scenarios. Without them, each scenario would be a deterministic LP. Therefore, in addition to the obvious modeling advantage, one can utilize a decomposition algorithm to exploit this sparse structure to achieve an optimal solution faster. Interior-point algorithms also tend to work faster on sparse constraint matrices (Lustig et al., 1991).

3.2. Structural Results

Before we attempt to analyze the problem numerically, we would like to point out some important remarks for the pricing problem we introduced above. Let $V_0^*\left(q^r, s_0\right)$ be the value of the contract with costless and uncapacitated storage and take-or-pay clause as in section 3.1. Then, the contract analyzed in section III.B of Thompson (1995) is a special case of our contract.

More specifically, if we disallow storage ($s_n = 0$ for $n = 0, \ldots, N$) and assume that the demand per period is equal to 1 unit ($d_n = 1$ for $n = 1, \ldots, N$), then

$$V_0^*\left(q^{ToP}, s_n = 0\right) = V_0^{no} - V_0^{T*}\left(q^{ToP}\right),$$

where $V_0^{T*}\left(q^{ToP}\right)$ is the value of the take-or-pay option as in Thompson (1995) and V_0^{no} is any convex combination of the price of the contract without any clauses and the cost of satisfying demand only in the spot market, that is

$$V_0^{no} = \alpha \times \left(\sum_{n=1}^{N} f_n d_n P_{0,n}\right) + \left(1-\alpha\right) \times \left(\sum_{j=1}^{J} \pi_j \sum_{n=1}^{N} f_n d_n P_{n,j}\right).$$

Next, we analyze the impact of the storage capability in the absence of a take-or-pay provision (i.e., the forward contract is enforced). More specifically, if we drop the take-or-pay provision and assume a contract only with costless and uncapacitated storage, the optimal strategy would depend on the sign of the convenience yield. If $\delta > 0$, it would be optimal to have zero storage for any period. The intuition is straightforward. As it is impossible to purchase in the spot market, we would be comparing the forward prices in the current period with those in the future periods. For $\delta > 0$, $P_{0,n} > PV^n(P_{0,n+1})$ simply because $P_0 e^{(r-\delta)n\Delta t} > P_0 e^{(r-\delta)(n+1)\Delta t} e^{-r\Delta t} = P_0 e^{(r-\delta)n\Delta t} e^{-\delta\Delta t}$. This implies that the value of such a contract is equal to $\sum_{n=1}^{N} f_n d_n P_{0,n} = \sum_{n=1}^{N} e^{-rn\Delta t} P_0 e^{(r-\delta)n\Delta t} d_n = P_0 \sum_{n=1}^{N} e^{-\delta n\Delta t} d_n$. This result would not change in the case of costly and limited storage. Similarly, if $\delta < 0$, it is optimal to purchase everything upfront and fully utilize the storage. In that case, the value of the contract is equal to $f_1 P_{0,1} \sum_{n=1}^{N} d_n = e^{-r\Delta t} P_0 e^{(r-\delta)\Delta t} \sum_{n=1}^{N} d_n = P_0 e^{-\delta\Delta t} \sum_{n=1}^{N} d_n$. If there are costs and limits for storage, then the optimal strategy decreases storage utilization and the contract value worsens (i.e., increases).

These results will be useful in analyzing the sub-additivity of the contract in section 4.2.

4. Computational Results

In this section, we solve the pricing problem using the multi-stage stochastic programming formulation introduced in section 3.1.2. As explained before, this approach yields a sparse A-matrix, which can be exploited by large-scale LP solvers through various decomposition algorithms. One downside of this formulation is that the problem size gets extremely large as the number of periods or scenarios increase. This is due to the exponential growth in the size of the scenario tree as a function of the number of periods or number of branches per period. This so-called *curse of dimensionality* has been less of a problem due to advances in hardware and software infrastructure.

In our base case, we consider a 4-quarter (= 1-year) supply chain contract[1] where the buyer has a take-or-pay option that expires at the end of the fourth quarter. We assume that the deterministic demand for each quarter is fully met by utilizing the local storage or new procurement. If new procurement is to be done, this would be carried out either in the market at the prevailing spot prices or through the contract at the forward prices that were set at the beginning of the first quarter. As such, the forward prices are deterministic whereas the spot prices are stochastic. We assume that the spot prices follow a Geometric Brownian Motion as follows:

$$\frac{dP}{P} = (r - \delta)\, dt + \sigma dW \tag{8}$$

Here, dW is a standard Wiener process, r is the risk-free rate, δ is the convenience yield net of storage costs, and σ is the volatility of spot prices. The constant parameters can be

[1] A 4-quarter supply chain contract can be also considered as a generic 4-period contract without loss of generality, where periods could be considered as one month to a year depending on the specific supply chain in focus.

made stochastic and modeled via a system of stochastic processes; however this would complicate the numerical approximation procedure and reduce the speed of convergence.

For the base case, we choose to adopt the parameter values in Thompson (1995), where possible, as our model is a generalization of the first type of contract in his work. Table 1 provides the full set of base case parameter values.

Table 1 **Parameter values for the base case**

Parameter	Value
Initial spot market price (P_0)	\$1.50/unit
Demand per quarter (d_n)	1 unit
Contract duration (T)	1 year
Take-or-pay percentage (φ)	50%
Spot price volatility (σ)	13%
Penalty cost multiplier (z)	100%
Storage cost multiplier (c^S)	0
Annual risk-free rate (r)	6%
Annual net convenience yield (δ)	2%
Storage capacity (u^S)	100%

It should be noted that the penalty cost parameter is set to be equal to the forward price for the final date times the penalty cost multiplier, that is $b = z * P_{0,N}$. Furthermore, we define, the take-or-pay requirement, q^{ToP}, as product of the total demand and the take-or-pay percentage, that is $q^{ToP} = \varphi * \sum_{n=1}^{N} d_n$.

Because a discrete-time scenario tree is required, we adopt the standard binomial lattice approach by Cox, Ross, and Rubinstein (1979) to implement (8). We divide each quarter into 15 time-steps, i.e., each time-step ($\Delta\tau$) is slightly less than a week. At every time-step, the price is multiplied by either $u = e^{\sigma\sqrt{\Delta\tau}}$ or $d = e^{-\sigma\sqrt{\Delta\tau}}$. Since decisions are to be made at the end of each quarter, we actually do not need the commodity prices in the interim time-steps. On the other hand, this time-granularization helps us enrich the representation of uncertainty and reduce the sampling error in the results. Our final four-period scenario tree, thus, has a multinomial structure where 16 branches (one more than the number of time-steps, due to the lattice) emanate out of each node. This results in 65,636 ($=16^4$) scenarios and 69905 nodes. We also obtain the scenario probabilities (π_j) as a by-product of this procedure. As we are pricing an option-embedded contract, we must use the risk-neutral probabilities for each branch on the tree to compute π_j. Based on the binomial lattice methodology, the risk-neutral probability of every price increase in a given time-step is computed as:

$$p^{RN} = \left(e^{(r-\delta)\Delta\tau} - d\right) \Big/ \left(u - d\right).$$

This base case setup leads to a large scale problem with close to 1.2 million variables and slightly more than 1 million equality constraints. However, the A-matrix is quite sparse with only about 2.7 million nonzero entries. Therefore, we are able to solve such a large problem in less than 5 minutes using Gurobi solver (version 5.6.3) on an Intel Core i7 workstation with 8 GB memory.

4.1. Costless and Uncapacitated Storage

4.1.1. Valuation of the Take-or-Pay Contract in the Presence of Spot Market. We first carry out univariate analyses to investigate the impact of market variables and take-or-pay contract specifications on the price of the contract. Figure 3 displays the contract value on the vertical axis as a function of these parameters. Interest rate parameter varies from 1% to 11%, whereas convenience yield changes between -3% and 7%. The volatility of gas prices is assumed to take values between 5% and 55%. Finally, we analyze the sensitivity of the contract price to contract terms: percentage of take-or-pay and the penalty cost as a percentage of the forward price. These parameters can take any value between 0 and 100%. For each analysis, all other variables take their base case values in Table 1.

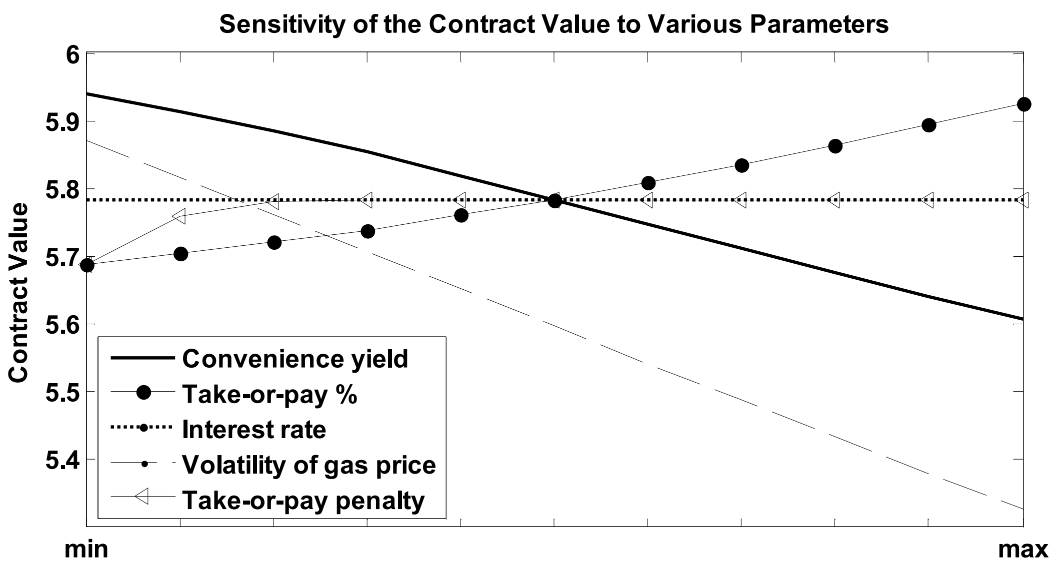

Figure 3 **Contract value vs various parameters when storage is costless and uncapacitated**

We observe that the contract price is most sensitive to the volatility of gas prices and the convenience yield. Higher volatility implies a higher option value for the take-or-pay characteristic of the contract and therefore reduces the price of the contract compared to a plain forward contract. A lower convenience yield results in a more expensive contract. Intuitively, a lower convenience yield (especially a negative one) means that the cost of storing natural gas embedded in the forward prices is higher. This will make the overall contract more expensive even though the local storage is costless and uncapacitated. The risk-free interest rate appears to have no impact on the value of the contract. This is due to the two conflicting but equal forces that are driven by the interest rates. One is the positive impact on the expected future gas prices and the other one is the negative effect on the present value of the future cash flows. These two forces appear to cancel each other within the value of the contract.

The contract value increases for higher take-or-pay percentage in a somewhat linear way. This is expected given that the value of the option disappears as the take-or-pay requirement increases, effectively reducing it to a forward. The impact of the take-or-pay penalty cost parameter, which is defined as a percentage of the final period's forward price, is not monotonic. Up to a certain level, a higher penalty cost implies a higher contract value. Beyond this level, the impact of the penalty cost levels off, implying that the take-or-pay percentage appears to be the dominant factor.

4.1.2. Contract Usage vis-à-vis Spot Market Trading. Next, we analyze how the aforementioned factors influence the optimal decisions. More specifically, we look into how the expected contract usage (not abandoning the forward contracts in favor of spot markets) changes as a function of these parameters and how, if any, the costless and uncapacitated storage is utilized. Figure 4 displays the average percentage of demand that is satisfied through the contract in the vertical axis as a univariate function of these parameters defined as above.

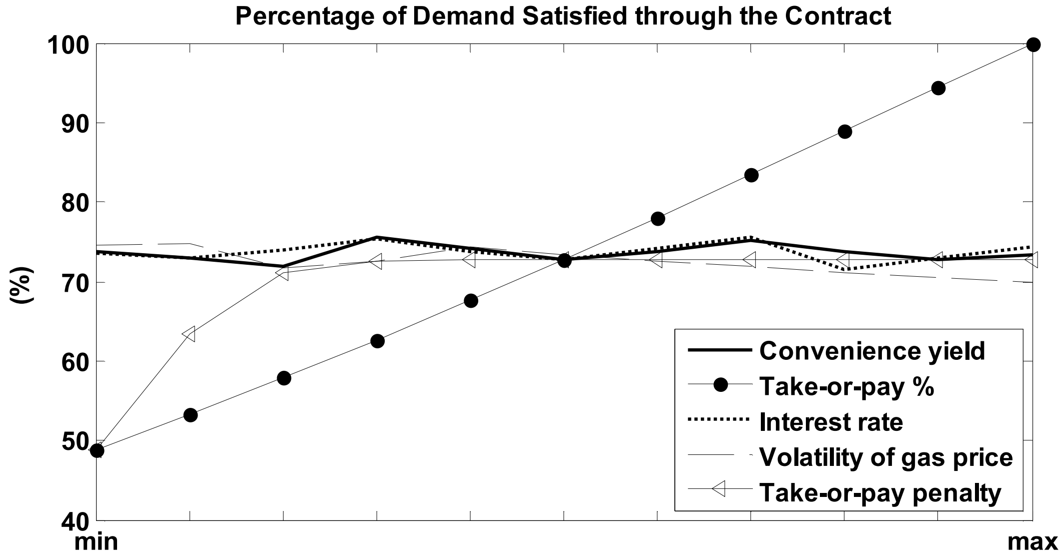

Figure 4 Average forward contract usage vs various parameters when storage is costless and uncapacitated

We observe that market variables (risk-free rate, volatility, and convenience yield) do not have a meaningful impact on the average contract usage. Nevertheless, increasing volatility beyond a certain level (25%) slightly decreases the forward contract usage and increases the chance of a penalty payment. This happens because the take-or-pay option becomes more valuable with higher volatility and early exercise becomes more likely.

For the relationship between contract usage and take-or-pay parameters, we can see a very similar pattern to Figure 3. In other words, take-or-pay percentage has an almost linear effect on the contract usage. Interestingly, average contract usage is almost always above the take-or-pay level except for the 100% case, which is equivalent to a fully enforced forward contract. Let's consider the case where take-or-pay percentage is zero. In this case, forward prices are favorable in the majority of the upper part of the scenario tree where spot prices are higher; therefore we observe almost 50% contract usage, which corresponds to the risk-neutral probability of spot prices being above the forward prices. In other words, when spot price is greater than the forward price at a given period, the forward contract is utilized. Therefore, the forward contract utilization ratio for a given period is the risk-neutral probability of the spot price to exceed the forward price for that period. If these risk-neutral probabilities are average over time, we find the average forward contract utilization, which turns out to be 48.86%.

4.1.3. Impact of Take-or-Pay Penalty Cost and Percentage Obligation. Take-or-pay penalty cost, on the other hand, has a positive but diminishing impact on the average contract usage, leveling off around 72.7%. In order to understand why the penalty cost has no impact on the contract value or the average contract usage, we plot how the likelihood of paying the penalty changes with respect to this parameter in Figure 5. Clearly, a penalty cost above 30% is so high that it will always be optimal to meet the take-or-pay percentage in order to avoid this penalty. Hence, without a take-or-pay violation, the contract value and the forward usage will always be the same for these values of the penalty cost.

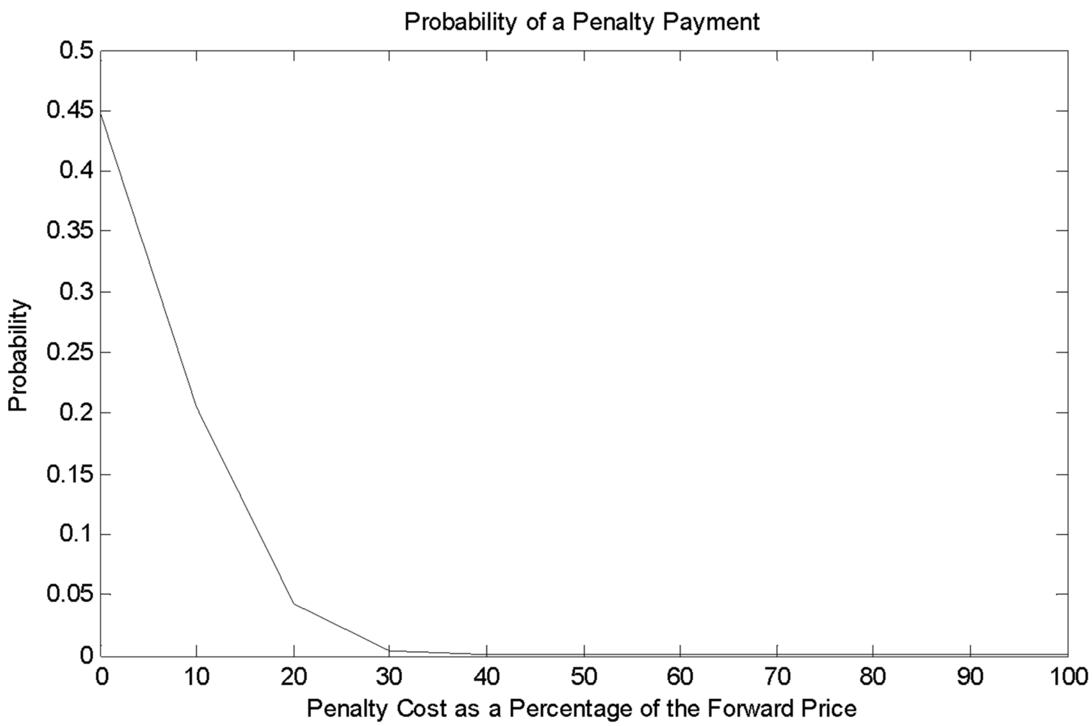

Figure 5 **Probability of paying a penalty vs cost of penalty when storage is costless and uncapacitated**

Next, we carry out a bivariate analysis to see if the diminishing level of the penalty cost also depends on the take-or-pay percentage. Figure 6 clearly shows that the contract becomes fully flexible when either parameter is zero. For any level of take-or-pay percentage, increasing the penalty cost beyond 30% has no impact on the contract value or forward usage; however, this plateau level increases with the take-or-pay percentage. At the extreme case of 100% take-or-pay percentage, when the penalty cost is low enough, we see that average forward usage can be less than 100%. However, the usage quickly becomes 100%, that is to say the supply chain contract becomes a portfolio of forward contracts, once the penalty cost increases beyond 30%. Regardless, in high take-or-pay levels, the benefit of the take-or-pay contract is relatively much higher compared to low take-or-pay levels. The reason for not observing any benefits in the case of high penalty costs is that the binomial lattice does not provide low enough spot prices to favor not purchasing through the contract and incurring penalties. For example, the probability of observing a spot price path that is low enough to beat the forward price path in case of a 20% penalty cost multiplier is 0.058. As expected, this value is equal to the probability of not satisfying the take-or-pay level of 100% and paying a penalty. This probability quickly diminishes to 0.0035 when the penalty cost is increased to 30% and becomes 0 when the cost multiplier reaches 70%. Thompson (1995) also documents a similar pattern. We note that this managerial insight can be used artfully while designing the penalty scheme into the contracts at the negotiation stage (Haksöz, 2013).

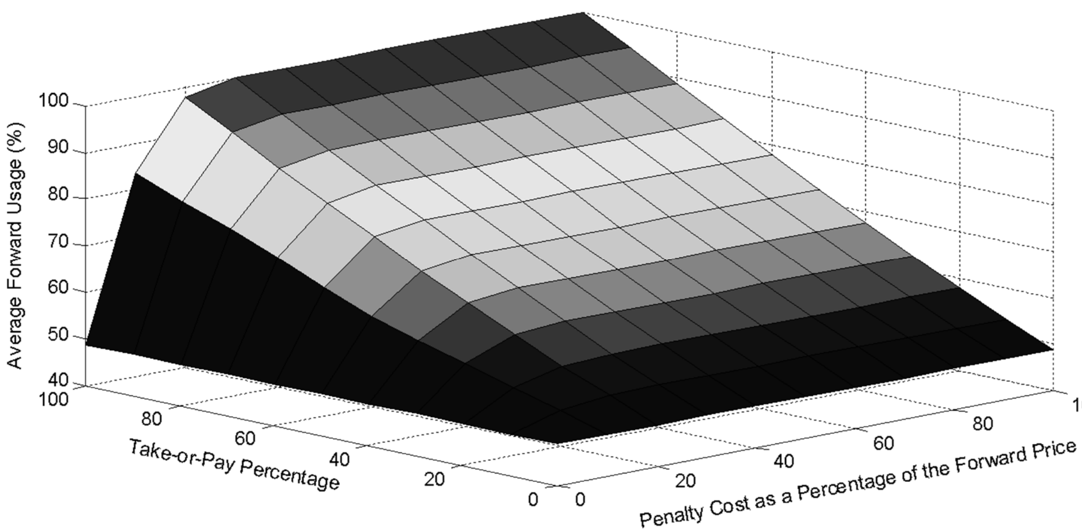

Figure 6 **Average forward contract usage vs contract parameters when storage is costless and uncapacitated**

4.1.4. Optimal Use of Storage Capacity. Finally, we look into the relationship between the five parameters and the utilization of costless and uncapacitated storage. In order to do this, we check whether storage is utilized under each scenario and compute the probability of those scenarios with positive storage. We find that the convenience yield is the only parameter that has an impact on the storage utilization. More specifically, when convenience yield is negative, probability of storage utilization becomes 1 and in any other case it is zero.[2] In order to understand the relationship between the convenience yield and

[2] The results are available with the authors upon request.

storage utilization, we further analyze how the expected number of units in storage varies across time for different values of this parameter (Figure 7).

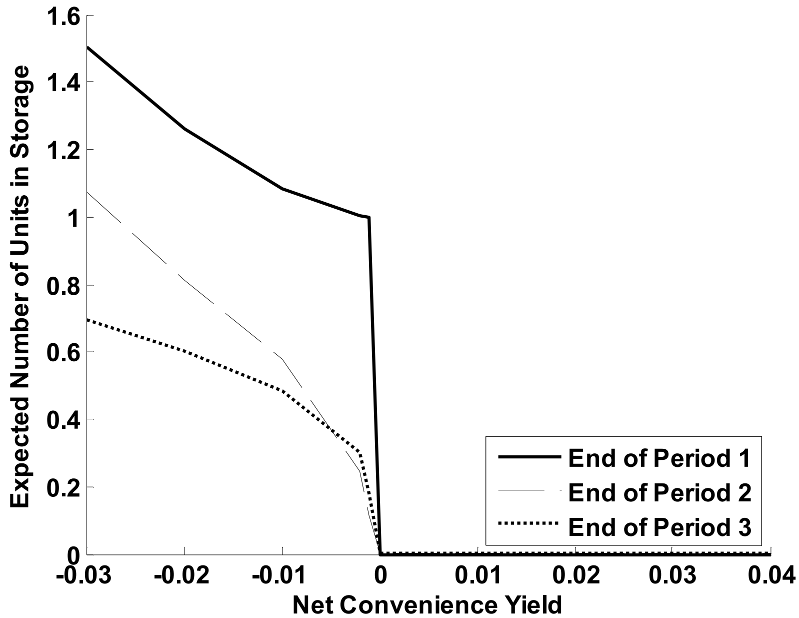

Figure 7 **Utilization of costless and uncapacitated storage versus net convenience yield**

We observe in Figure 7 that the storage is never utilized for positive values of the convenience yield. When net convenience yield becomes negative, storage becomes a valuable option and is significantly utilized in the early stages of the contract duration. More negative net convenience yield is equivalent to higher implied storage costs and therefore results in higher utilization of costless storage. Although the chart shows positive storage values for 0 convenience yield, this is actually an arbitrary value as any storage solution gives the same contract value (i.e., multiple optima). This phenomenon is explained by the nature of convenience yield, which actually is defined net of storage costs in our paper.

A negative convenience yield suggests that implied storage cost charged by the seller is too high (even higher than the risk-free rate). This makes the expected present value of the future spot prices (and forward prices) higher than the current spot prices. Therefore, it becomes optimal to buy sooner to meet the future demand as the local storage is defined to be costless and unlimited.

On the other hand, when the convenience yield is positive, the implied storage cost is relatively cheap and even less than the gross convenience yield, rendering current spot and forward prices unfavorable. Therefore, it becomes optimal to meet the demand as it arrives as opposed to utilizing the local storage even though it is free. This result states that the convenience yield is one of the strategic levers that should be constantly monitored and interpreted correctly in natural gas procurement and trading.

4.2. Costly and Limited Storage

In this section, we focus on the cases where the net convenience yield is less than 0 as the storage would not be utilized otherwise even when it is costless and uncapacitated. We first modify the objective function (4) of the stochastic program in section 3.1.2 to reflect the costly storage:

$$\min \quad \sum_{j=1}^{J} \pi_j \left(\sum_{n=1}^{N} f_n \left(P_{0,n} q_{n,j}^{C} + P_{n,j} q_{n,j}^{M} \right) + f_N \left(b \left(q^{ToP} - \sum_{n=1}^{N} q_{n,j}^{C} \right)^{+} \right) + \sum_{n=1}^{N-1} f_n \left(c^S P_{0,n} s_{n,j} \right) \right) \tag{9}$$

Next, we add the upper bound on the storage as a percentage of the total demand after the first period.

$$s_{n,j} \leq u^S * \sum_{n=2}^{N} d_n \tag{10}$$

The storage limit, u^S, varies from 100% of the total demand (i.e., uncapacitated) to 0% (no storage). Cost multiplier, c^S, varies from 0 (costless) to 0.006. This latter value is found by trial-and-error to be sufficiently high enough to discourage use of storage. Figure 8 displays how the contract value changes with respect to these two parameters for two different values of the net convenience yield. On the left, the implied storage costs (charged by the seller) are relatively too high compared to the value of holding the commodity for consumption. Therefore, costly storage is utilized as soon as the limit is relaxed from zero level. Of course, this utilization is reduced (and the contract value deteriorates) as the storage cost increases, but the benefits of local storage are visible. On the right, the net convenience yield is still negative but closer to zero and the benefits of storage has already begun to diminish. In this case, a storage cost multiplier of 0.003 is sufficient to discourage the use of local storage regardless of the capacity. Therefore, the contract is often reduced to a standard take-or-pay.

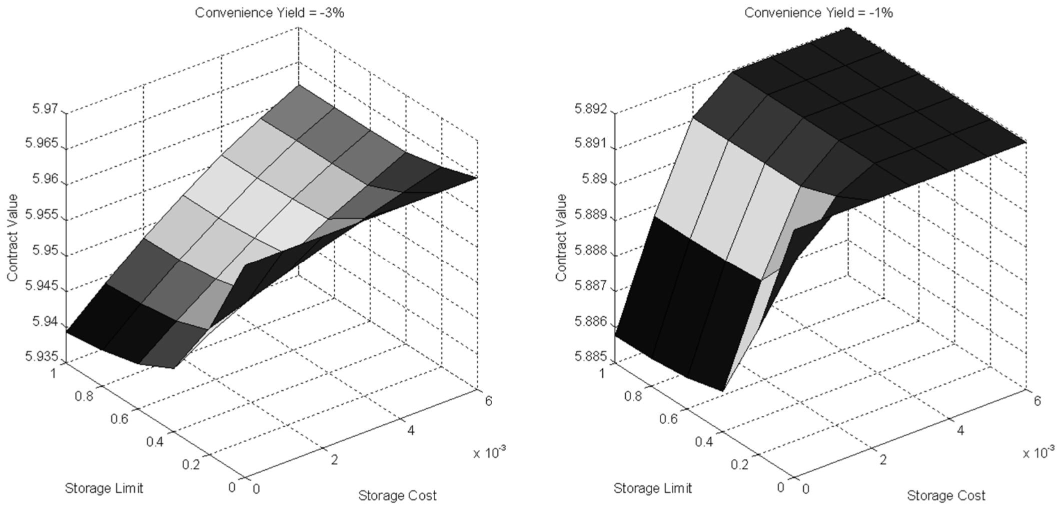

Figure 8 **Contract value vs storage parameters**

We have so far shown that the presence of local storage enhances the value of the procurement contract we introduced in section 3. However, if $\delta > 0$, we have observed that regardless of the presence of a take-or-pay clause, $V_0^*\left(q^r, s_0\right) = V_0^*\left(q^r, s_t = 0\right)$. That is to say, storage is not utilized even when it is free and unlimited.

4.2.1. Sub-additivity of Option Value. We next attempt to quantify the sub-additivity of these two options for the case $\delta < 0$. In other words, the decrease in the value of the contract due to take-or-pay option and storage utilization turns out to be less than (or equal to) sum of the value of the take-or-pay option and that of the storage. Option value sub-additivity is shown for abandonment and price renegotiation options in a bundled option structure by Haksöz and Simsek (2010). On the other hand, both Trigeorgis (1993) and Agliardi (2006, 2007) demonstrate that option interactions may result in super-additivity under certain conditions. Hence, due to emergent structure of option interactions, it is not possible *ex ante* to determine if the combined option value is sub vs. super-additivity. In our case, Figure 9 shows that the loss due to the sub-additivity is greater when convenience yield is more negative. Furthermore, the storage capacity appears to be a more important determinant of this loss compared to cost of storage. For unlimited and costless storage (top left corners), the sub-additivity can result in a loss of 10% to 30% of the value of the take-or-pay option; however, we still have the cheapest contract in this case as shown in Figure 8.

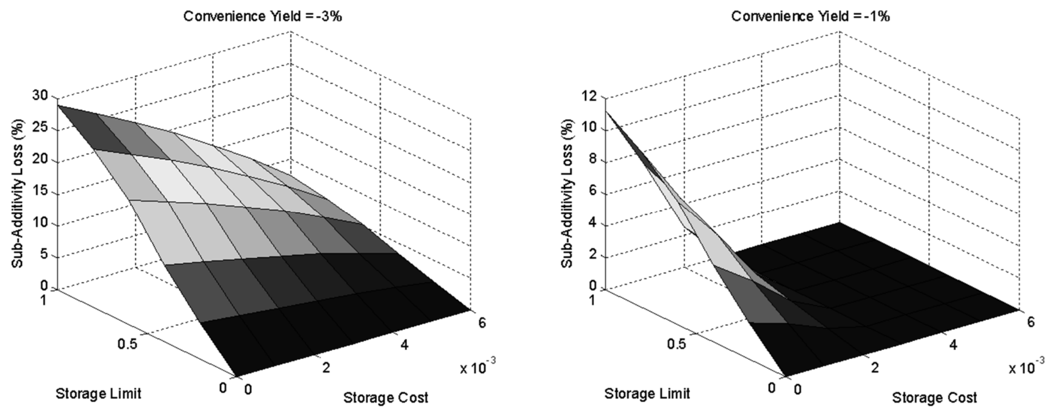

Figure 9 **Loss due to sub-additivity of storage and take-or-pay vs storage parameters**

5. Conclusions

In this paper, we provide a beneficial and practically relevant analysis of a take-or-pay contract in the presence of spot market and storage capability. We employ the multi-stage stochastic programming method in order to explicitly consider various interacting managerial levers. Since this study is motivated by the natural gas procurement, managerial insights generated would help decision makers in natural gas as well as other commodity supply chains to better design, plan and act where spot market price and storage uncertainties abound.

Solving the multi-stage stochastic program in a dynamic setting by split-variable formulation, we demonstrate the impact of major managerial levers on take-or-pay contract design, valuation, and usage. In sum, take-or-pay contract penalty cost, take-or-pay percentage obligation, net convenience yield, and storage capacity are shown to play key roles in this process. Moreover, we numerically show the sub-additivity of the option value, i.e., take-or-pay and storage options, such that bundling both options may increase the total value less than the sum of individual values, which is an important insight in option portfolio construction.

Our work contributes to amalgamating separate streams of previous research conducted in supply chain commodity procurement and valuation of multiple interacting options in a take-or-pay contract framework. To this end, we do hope that our paper becomes a first step in analyzing the real world interactions of take-or-pay contracts, spot market trading, as well as storage capabilities as they occur in practice without neglecting the necessary rigorous analysis.

Appendix. DP Formulation for Costless and Uncapacitated Storage

We begin from the very last period, denoted as $N+1$ and move backwards in time.

$$V_{N+1}(q^{ToP}, s_{N+1}) = b.(q^{ToP})^{+} \tag{11}$$

$$\begin{aligned} V_N(q^{ToP}, s_N) &= E\min P_{0,N} q_N^C + P_N q_N^M + V_{N+1}(q^r - q_N^C, 0) \\ q_N^C + q_N^M &= d_N - s_N \\ 0 \le s_N &\le d_N \end{aligned} \tag{12}$$

$$\begin{aligned} V_{N-1}(q^{ToP}, s_{N-1}) &= E\min P_{0,N-1} q_{N-1}^C + P_{N-1} q_{N-1}^M + e^{-r} V_N(q^{ToP} - q_{N-1}^C, s_{N-1} + q_{N-1}^M + q_{N-1}^C - d_{N-1}). \\ \left(d_{N-1} + s_{N-1}\right)^{+} &\le q_{N-1}^C + q_{N-1}^M \le d_{N-1} + d_N - s_{N-1}, \\ 0 \le s_{N-1} &\le d_{N-1} + d_N. \end{aligned} \tag{13}$$

$$\begin{aligned} V_{N-2}(q^{ToP}, s_{N-2}) &= E\min P_{0,N-2} q_{N-2}^C + P_{N-2} q_{N-2}^M + e^{-r} V_{N-1}(q^{ToP} - q_{N-2}^C, s_{N-2} + q_{N-2}^M + q_{N-2}^C - d_{N-2}) \\ \left(d_{N-2} + s_{N-2}\right)^{+} &\le q_{N-2}^C + q_{N-2}^M \le d_{N-2} + d_{N-1} + d_N - s_{N-1}, \\ 0 \le s_{N-2} &\le d_{N-2} + d_{N-1} + d_N. \end{aligned} \tag{14}$$

First, we begin solving the value function at the end of the horizon. The value function, $V_N(q^{ToP}, s_N)$ can be expressed as follows:

$$V_N(q^{ToP}, s_N) = E\min P_{0,N} q_N^C + P_N q_N^M + b(q^{ToP} - q_N^C)^{+}$$

We have three cases based on the value of q^{ToP}. Thus, we have

Case 1. If $q^{ToP} > d_N - s_N$, then

$$P_{0,N} \le P_N + b \ \Rightarrow q_N^C = d_N - s_N, q_N^M = 0.$$

$$P_{0,N} > P_N + b \ \Rightarrow q_N^C = 0, q_N^M = d_N - s_N.$$

Case 2. If $0 < q^{ToP} \le d_N - s_N$, then

$$P_{0,N} > P_N + b \Rightarrow q_N^C = 0, q_N^M = d_N - s_N.$$

$$P_N + b \geqslant P_{0,N} > P_N \Rightarrow q_N^C = q^{ToP}, q_N^M = d_N - s_N - q^{ToP}.$$

$$P_N > P_{0,N} \Rightarrow q_N^C = d_N - s_N, q_N^M = 0.$$

Case 3. If $q^{ToP} \le 0$, then

$$P_{0,N} \le P_N \Rightarrow q_N^C = d_N - s_N, q_N^M = 0.$$

$$P_{0,N} > P_N \Rightarrow q_N^C = 0, q_N^M = d_N - s_N .$$

Now, we analyze Case 1 in detail to identify structural properties of the value function. The value function, $V_N(q^{ToP}, s_N)$ can be written as follows:

$$\begin{aligned} V_N(q^{ToP}, s_N) &= E \min P_{0,N}(d_N - s_N) + b(q^{ToP} - (d_N - s_N)), P_N(d_N - s_N) + bq^{ToP}. \\ &= bq^{ToP} + E \min\left\{(P_{0,N} - b)(d_N - s_N), P_N(d_N - s_N)\right\}. \\ &= bq^{ToP} + (d_N - s_N)E\left((P_{0,N} - b) \wedge P_N\right). \end{aligned} \tag{15}$$

This implies that the value function $V_N(q^{ToP}, s_N)$ is linear in (q^{ToP}, s_N).

Now, let us express the value function at time period $N-1$. We obtain

$$\begin{aligned} V_{N-1}(q^{ToP}, s_{N-1}) &= E \min\Big[P_{0,N-1}q_{N-1}^C + P_{N-1}q_{N-1}^M \\ &\quad + e^{-r}V_N(q^{ToP} - q_{N-1}^C, s_{N-1} + q_{N-1}^M + q_{N-1}^C - d_{N-1})\Big]. \\ &= E \min\Big[P_{0,N-1}q_{N-1}^C + P_{N-1}q_{N-1}^M + e^{-r}b(q^{ToP} - q_{N-1}^C) \\ &\quad + e^{-r}(d_N - s_{N-1} - q_{N-1}^C - q_{N-1}^M + d_{N-1})E\left((P_{0,N} - b) \wedge P_N\right)\Big]. \end{aligned}$$

This can be further written as follows by denoting the expected value $E\left((P_{0,N} - b) \wedge P_N\right)$ as $E(.)$ to simplify the expressions.

$$\begin{aligned} V_{N-1}(q^{ToP}, s_{N-1}) &= e^{-r}bq^{ToP} + E \min\Big[q_{N-1}^C\{P_{0,N-1} - e^{-r}b - e^{-r}E(.)\} + q_{N-1}^M\{P_{N-1} - e^{-r}E(.)\} \\ &\quad + e^{-r}(d_N - s_{N-1} + d_{N-1})E(.)\Big]. \end{aligned}$$

We can now characterize the optimal procurement quantities of the contract and the spot market using this expression with the constraints given in (13). We have two cases.

Case 1. $\min\left\{P_{0,N-1} - e^{-r}b - e^{-r}E(.), P_{N-1} - e^{-r}E(.)\right\} < 0$, then

$$P_{0,N-1} - e^{-r}b \le P_{N-1} \Rightarrow q_{N-1}^C = d_{N-1} - s_{N-1} + d_{N,}\ q_{N-1}^M = 0.$$

$$P_{0,N-1} - e^{-r}b > P_{N-1} \Rightarrow q_{N-1}^C = 0, q_{N-1}^M = d_{N-1} - s_{N-1} + d_N .$$

Case 2. $\min\left\{P_{0,N-1} - e^{-r}b - e^{-r}E(.), P_{N-1} - e^{-r}E(.)\right\} \ge 0$, then

$$P_{0,N-1} - e^{-r}b \le P_{N-1} \Rightarrow q_{N-1}^C = (d_{N-1} - s_{N-1})^+, q_{N-1}^M = 0.$$

$$P_{0,N-1} - e^{-r}b > P_{N-1} \Rightarrow q_{N-1}^C = 0, q_{N-1}^M = (d_{N-1} - s_{N-1})^+ .$$

These results imply that the value function $V_{N-1}(q^{ToP}, s_{N-1})$ is piecewise linear in (q^{ToP}, s_{N-1}).

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